Babylonian Mathematics[1]

1 Introduction

Our first knowledge of mankind's use of mathematics comes from the Egyptians and Babylonians. Both civilizations developed mathematics that was similar in scope but different in particulars. There can be no denying the fact that the totality of their mathematics was profoundly elementary[2], but their astronomy of later times did achieve a level comparable to the Greeks.

2 Basic Facts

The Babylonian civilization has its roots dating to 4000BCE with the Sumerians in Mesopotamia. Yet little is known about the Sumerians. Sumer was first settled between 4500 and 4000 BC by a non-Semitic

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[1] ©2002, G. Donald Allen
[2] Neugebauer, 1951

people who did not speak the Sumerian language. These people now are called Ubaidians, for the village Al-Ubaid, where their remains were first uncovered. Even less is known about their mathematics. Of the little that is known, the Sumerians of the Mesopotamian valley built homes and temples and decorated them with artistic pottery and mosaics in geometric patterns. The Ubaidians were the first civilizing force in the region. They drained marshes for agriculture, developed trade and established industries including weaving, leatherwork, metalwork, masonry, and pottery. The people called Sumerians, whose language prevailed in the territory, probably came from around Anatolia, probably arriving in Sumer about 3300 BC. For a brief chronological outline of Mesopotamia see

http://www.gatewaystobabylon.com/introduction/briefchonology.htm. See also http://www.wsu.edu:8080/~dee/MESO/TIMELINE.HTM for more detailed information.

The early Sumerians did have writing for numbers as shown below. Owing to the scarcity of resources, the Sumerians adapted the ubiquitous clay in the region developing a writing that required the use of a stylus to carve into a soft clay tablet. It predated the

| 1 | 10 | 60 | 600 | 3,600 | 36,000 |
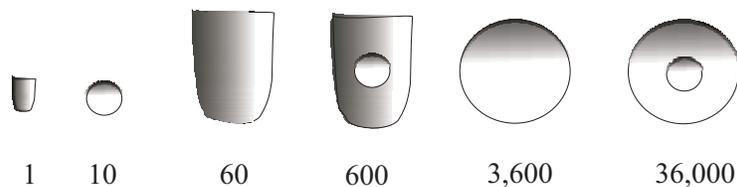
cuneiform (wedge) pattern of writing that the Sumerians had developed during the fourth millennium. It probably antedates the Egyptian hieroglyphic may have been the earliest form of written communication. The Babylonians, and other cultures including the Assyrians, and Hittites, inherited Sumerian law and literature and importantly their style of writing. Here we focus on the later period of the Mesopotamian civilization which engulfed the Sumerian civilization. The Mesopotamian civilizations are often called Babylonian, though this is not correct. Actually, Babylon[3] was not the first great city, though the whole civilization is called *Babylonian*. Babylon, even during its existence, was not always

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[3]The first reference to the Babylon site of a temple occurs in about 2200 BCE. The name means "gate of God." It became an independent city-state in 1894 BCE and Babylonia was the surrounding area. Its location is about 56 miles south of modern Baghdad.

the center of Mesopotamian culture. The region, at least that between the two rivers, the Tigris and the Euphrates, is also called Chaldea.

The dates of the Mesopotamian civilizations date from 2000-600 BCE. Somewhat earlier we see the unification of local principates by powerful leaders — not unlike that in China. One of the most powerful was **Sargon** the Great (ca. 2276-2221 BC). Under his rule the region was forged into an empire called the dynasty of Akkad and the Akkadian language began to replace Sumerian. Vast public works, such as irrigation canals and embankment fortifications, were completed about this time. These were needed because of the nature of the geography combined with the need to feed a large population. Because the Trigris and Euphrates would flood in heavy rains and the clay soil was not very absorptive, such constructions were necessary if a large civilization was to flourish.

Later in about 2218 BCE tribesmen from the eastern hills, the Gutians, overthrew Akkadian rule giving rise to the 3rd Dynasty of Ur. They ruled much of Mesopotamia. However, this dynasty was soon to perish by the influx of Elamites from the north, which eventually destroyed the city of Ur in about 2000 BC. These tribes took command of all the ancient cities and mixed with the local people. No city gained overall control until Hammurabi of Babylon (reigned about 1792-1750 BCE) united the country for a few years toward the end of his reign.

The Babylonian "texts" come to us in the form of clay tablets, usually about the size of a hand. They were inscribed in cuneiform, a wedge-shaped writing owing its appearance to the stylus that was used to make it. Two types of mathematical tablets are generally found, *table-texts* and *problem texts*. Table-texts are just that, tables of values for some purpose, such as multiplication tables, weights and measures tables, reciprocal tables, and the like. Many of the table texts are clearly "school texts", written by apprentice scribes. The second class of tablets are concerned with the solutions or methods of solution to algebraic or geometrical problems. Some tables contain up to two hundred problems, of gradual increasing difficulty. No doubt, the role of the teacher was significant.

Babylon fell to **Cyrus of Persia** in 538 BC, but the city was spared.
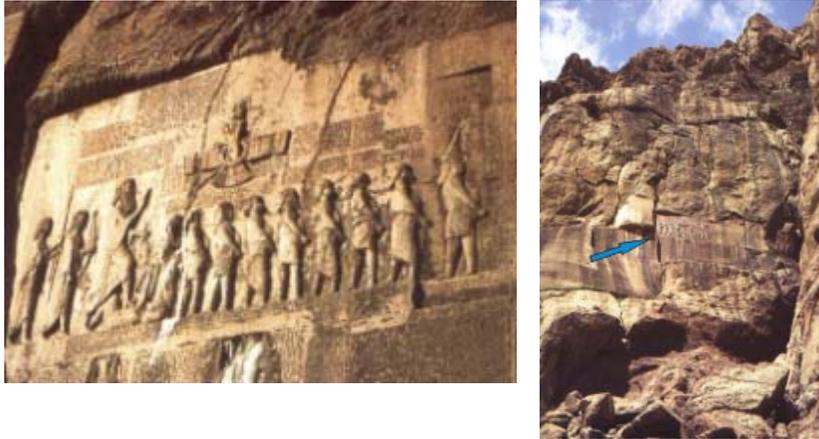
The Darius inscription on cliff near Bisotun

The great empire was finished. However, another period of Babylonian mathematical history occurred in about 300BCE, when the Seleucids, successors of Alexander the Great came into command. The 300 year period has furnished a great number of astronomical records which are remarkably mathematical — comparable to Ptolemy's *Almagest*. Mathematical texts though are rare from this period. This points to the acuity and survival of the mathematical texts from the old-Babylonian period (about 1800 to 1600 BCE), and it is the old period we will focus on.

The use of cuneiform script formed a strong bond. Laws, tax accounts, stories, school lessons, personal letters were impressed on soft clay tablets and then were baked in the hot sun or in ovens. From one region, the site of ancient Nippur, there have been recovered some 50,000 tablets. Many university libraries have large collections of cuneiform tablets. The largest collections from the Nippur excavations, for example, are to be found at Philadelphia, Jena, and Istanbul. All total, at least 500,000 tablets have been recovered to date. Even still, it is estimated that the vast bulk of existing tablets is still buried in the ruins of ancient cities.

Deciphering cuneiform succeeded the Egyptian hieroglyphic. Indeed, just as for hieroglyphics, the key to deciphering was a trilingual inscription found by a British office, Henry Rawlinson (1810-1895), stationed as an advisor to the Shah. In 516 BCE Darius the Great, who reigned in 522-486 BCE, caused a lasting monument[4] to his rule to be engraved in bas relief on a 100×150 foot surface on a rock cliff, the "Mountain of the Gods" at Behistun[5] at the foot of the Zagros Mountains in the Kermanshah region of modern Iran along the road between modern Hamadan (Iran) and Baghdad, near the town of Bisotun. In antiquity, the name of the village was Bagastâna, which means 'place where the gods dwell'.

Like the Rosetta stone, it was inscribed in three languages — Old Persian, Elamite, and Akkadian (Babylonian). However, all three were then unknown. Only because Old Persian has only 43 signs and had been the subject of serious investigation since the beginning of the century was the deciphering possible. Progress was very slow. Rawlinson was able to correctly assign correct values to 246 characters, and moreover, he discovered that the same sign could stand for different consonantal sounds, depending on the vowel that followed. (*polyphony*) It has only been in the 20th century that substantial publications have appeared. Rawlinson published the completed translation and grammar in 1846-1851. He was eventually knighted and served in parliament (1858, 1865-68).

For more details on this inscription, see the article by Jona Lendering at
http://www.livius.org/be-bm/behistun/behistun01.html. A translation is included.

3   Babylonian Numbers

In mathematics, the Babylonians (Sumerians) were somewhat more advanced than the Egyptians.

- Their mathematical notation was positional but sexagesimal.

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[4]According to some sources, the actual events described in the monument took place between 522 and 520 BCE.

[5]also spelled Bistoun

- They used no zero.
- More general fractions, though not all fractions, were admitted.
- They could extract square roots.
- They could solve linear systems.
- They worked with Pythagorean triples.
- They solved cubic equations with the help of tables.
- They studied circular measurement.
- Their geometry was sometimes incorrect.

For enumeration the Babylonians used symbols for 1, 10, 60, 600, 3,600, 36,000, and 216,000, similar to the earlier period. Below are four of the symbols. They did arithmetic in base 60, *sexagesimal*.

1   10   60   600

Cuneiform numerals
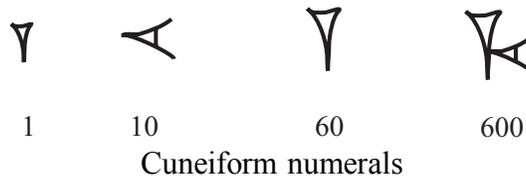
For our purposes we will use just the first two symbols

$$\vee \;=\; 1 \qquad \prec \;=\; 10$$

All numbers will be formed from these.

Example:

$$\frac{\prec\prec \;\; \vee\vee\vee}{\prec\prec\prec \vee\vee\vee\vee} = 57$$

Note the notation was **positional** and **sexagesimal**:

$$\prec\prec \;\; \prec\prec = 20 \cdot 60 + 20$$

$$\vee\vee \;\; \vee\vee \prec \vee = 2 \cdot 60^2 + 2 \cdot 60 + 21 = 7,331$$

The story is a little more complicated. A few shortcuts or abbreviation were allowed, many originating in the Seleucid period. Other

devices for representing numbers were used. Below see how the number 19 was expressed.

Three ways to express the number 19

≪ᛉ = 19   Old Babylonian. The symbol ⌐ means subtraction.

⟨⫲ = 19   Formal

⟨⫸ = 19   Cursive form
            Seleucid Period(c. 320 BC to c. 620 AD)

The horizontal symbol above the "1" designated subtraction.

There is no clear reason why the Babylonians selected the sexagesimal system[6] . It was possibly selected in the interest of **metrology**, this according to Theon of Alexandria, a commentator of the fourth century A.D.: i.e. the values 2,3,5,10,12,15,20, and 30 all divide 60. Remnants still exist today with time and angular measurement. However, a number of theories have been posited for the Babylonians choosing the base of 60. For example[7]

1. The number of days, 360, in a year gave rise to the subdivision of the circle into 360 degrees, and that the chord of one sixth of a circle is equal to the radius gave rise to a natural division of the circle into six equal parts. This in turn made 60 a natural unit of counting. (Moritz Cantor, 1880)

2. The Babylonians used a 12 hour clock, with 60 minute hours. That is, two of our minutes is one minute for the Babylonians. (Lehmann-Haupt, 1889) Moreover, the (Mesopotamian) zodiac was divided into twelve equal sectors of 30 degrees each.

3. The base 60 provided a convenient way to express fractions from a variety of systems as may be needed in conversion of weights and measures. In the Egyptian system, we have seen the values 1/1, 1/2, 2/3, 1, 2, . . . , 10. Combining we see the factor of 6 needed in the denominator of fractions. This with the base 10 gives 60 as the base of the new system. (Neugebauer, 1927)

4. The number 60 is the product of the number of planets (5 known at the time) by the number of months in the year, 12. (D. J. Boorstin,

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[6]Recall, the very early use of the sexagesimal system in China. There may well be a connection.
[7]See Georges Ifrah, *The Universal History of Numbers*, Wiley, New York, 2000.

1986)

5. The combination of the duodecimal system (base 12) and the base
    10 system leads naturally to a base 60 system. Moreover, duodeci-
    mal systems have their remnants even today where we count some
    commodities such as eggs by the dozen. The English system of
    fluid measurement has numerous base twelve values. As we see
    in the charts below, the base twelve (base 3, 6?) and base two
    graduations are mixed. Similar values exist in the ancient Roman,
    Sumerian, and Assyrian measurements.

	teaspoon	tablespoon	fluid ounce
1 teaspoon =	1	1/3	1/6
1 tablespoon =	3	1	1/2
1 fluid ounce =	6	2	1
1 gill =	24	8	4
1 cup =	48	16	8
1 pint =	96	32	16
1 quart =	192	64	32
1 gallon =	768	256	128
1 firkin =	6912	2304	1152
1 hogshead =	48384	16128	8064

	inch	foot	yard
1 inch =	1	1/12	1/36
1 foot =	12	1	1/3
1 yard =	36	3	1
1 mile =	- - -	5280	1760

Note that missing in the first column of the liquid/dry measurement
table is the important cooking measure 1/4 cup, which equals 12
teaspoons.

6. The explanations above have the common factor of attempting to
    give a plausibility argument based on some particular aspect of
    their society. Having witnessed various systems evolve in modern
    times, we are tempted to conjecture that a certain arbitrariness may
    be at work. To create or impose a number system and make it apply
    to an entire civilization must have been the work of a political
    system of great power and centralization. (We need only consider
    the failed American attempt to go metric beginning in 1971. See,
    http://lamar.colostate.edu/ hillger/dates.htm) The decision to adapt

the base may have been may been made by a ruler with little more than the advice merchants or generals with some vested need. Alternatively, with the consolidation of power in Sumeria, there may have been competing systems of measurement. Perhaps, the base 60 was chosen as a compromise.

Because of the large base, multiplication was carried out with the aide of a table. Yet, there is no table of such a magnitude. Instead there are tables up to 20 and then selected values greater (i.e. 30, 40, and 50). The practitioner would be expected to decompose the number into a sum of smaller numbers and use multiplicative distributivity.

A positional fault??? Which is it?

$$
\begin{aligned}
\prec\prec &= 10 \cdot 60 + 10 \\
&= 10 \cdot 60^2 + 10 = 3,610 \\
&= 10 + \frac{10}{60} \\
&= 20 (???)
\end{aligned}
$$

1. There is no "gap" designator.

2. There is a true floating point — its location is undetermined except from context.

$\star$ The "gap" problem was overcome in the Seleucid period with the invention of a "zero" as a gap separator.

We use the notation:

$$
d_1; d_2, d_3, \ldots = d_1 + \frac{d_2}{60} + \frac{d_3}{60^2} + \cdots
$$

The values $d_1; d_2, d_3, d_4, \ldots$ are all integers.

Example

$$
\begin{aligned}
1; 24, 51, 10 &= 1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} \\
&= 1.41421296
\end{aligned}
$$

This number was found on the Old Babylonian Tablet (Yale Collection #7289) and is a very high precision estimate of $\sqrt{2}$. We will continue this discussion shortly, conjecturing on how such precision may have been obtained.

The exact value of $\sqrt{2}$, to 8 decimal places is $= 1.41421356$.

Fractions. Generally the only fractions permitted were such as

$$\frac{2}{60}, \ \frac{3}{60}, \ \frac{5}{60}, \ \frac{12}{60}, \ \cdots$$

because the sexagesimal expression was known. For example,

$$\frac{1}{6} = \frac{10}{60} = ; \prec$$

$$\frac{1}{9} = ; \begin{matrix} \vee\vee\vee & \prec\prec \\ \vee\vee\vee & \prec\prec \end{matrix}$$

Irregular fractions such as $\frac{1}{7}, \frac{1}{11}$, etc were not normally not used. There are some tablets that remark, "7 does not divide", or "11 does not divide", etc.

A table of all products equal to sixty has been found.

2	30	16	3, 45
3	20	18	3,20
4	15	20	3
5	12	24	2,30
6	10	25	2,25
8	7,30	27	2,13,20
9	6,40	30	2
10	6	32	1;52,30
12	5	36	1,40
15	4	40	1,30

You can see, for example that

$$8 \times 7; 30 = 8 \times (7 + \frac{30}{60}) = 60$$

Note that we did not use the separatrix ";" here. This is because the table is also used for reciprocals. Thus

$$\frac{1}{8} = 0; 7, 30 = \frac{7}{60} + \frac{30}{60^2}$$

Contextual interpretation was critical.

Remark. The corresponding table for our decimal system is shown below. Included also are the columns with 1 and the base 10. The product relation and the decimal expansion relations are valid in base 10.

1	10
2	5
5	2
10	1

Two tablets found in 1854 at Senkerah on the Euphrates date from 2000 B.C. They give squares of the numbers up to 59 and cubes up to 32. The Babylonians used the formula

$$xy = ((x + y)^2 - (x - y)^2)/4$$

to assist in multiplication. Division relied on multiplication, i.e.

$$\frac{x}{y} = x \cdot \frac{1}{y}$$

There apparently was no long division.

The Babylonians knew some approximations of irregular fractions.

$$\frac{1}{59} =; 1, 1, 1 \qquad \frac{1}{61} =; 0, 59, 0, 59$$

However, they do not appear to have noticed infinite periodic expansions.[8]

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[8] In the decimal system, the analogous values are $\frac{1}{9} = 0.1111\ldots$ and $\frac{1}{11} = 0.090909\ldots$. Note the use of the units "0" here but not for the sexagesimal. Why?

They also seemed to have an elementary knowledge of logarithms. That is to say there are texts which concern the determination of the exponents of given numbers.

4   Babylonian Algebra

In Greek mathematics there is a clear distinction between the geometric and algebraic. Overwhelmingly, the Greeks assumed a geometric position wherever possible. Only in the later work of Diophantus do we see algebraic methods of significance. On the other hand, the Babylonians assumed just as definitely, an algebraic viewpoint. They allowed operations that were forbidden in Greek mathematics and even later until the 16$^\text{th}$ century of our own era. For example, they would freely multiply areas and lengths, demonstrating that the *units* were of less importance. Their methods of designating unknowns, however, does invoke units. First, mathematical expression was strictly rhetorical, symbolism would not come for another two millenia with Diophantus, and then not significantly until Vieta in the 16$^\text{th}$ century. For example, the designation of the unknown was *length*. The designation of the square of the unknown was *area*. In solving linear systems of two dimensions, the unknowns were *length* and *breadth*, and *length*, *breadth*, and *width* for three dimensions.

Square Roots.   Recall the approximation of $\sqrt{2}$. How did they get it? There are two possibilities: (1) Applying the method of the mean. (2) Applying the approximation

$$\sqrt{a^2 \pm b} \approx a \pm \frac{b}{2a}$$

Yale Babylonian Collection
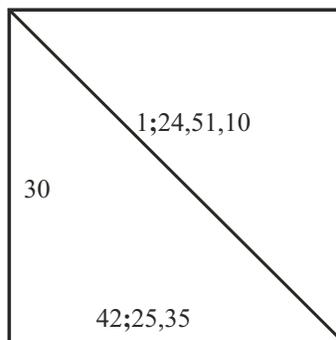
Square with side 30

The product of 30 by 1;24,51,10 is precisely 42;25,35.

Method of the mean. The method of the mean can easily be used to find the square root of any number. The idea is simple: to find the square root of 2, say, select x as a first approximation and take for another $2/x$. The product of the two numbers is of course 2, and moreover, one must be less than and the other greater than 2. Take the arithmetic average to get a value closer to $\sqrt{2}$. Precisely, we have

1. Take $a = a_1$ as an initial approximation.
2. Idea: If $a_1 < \sqrt{2}$ then $\frac{2}{a_1} > \sqrt{2}$.

3. So take
$$a_2 = (a_1 + \frac{2}{a_1})/2.$$

4. Repeat the process.

Example. Take $a_1 = 1$. Then we have

$$\begin{array}{rcl}
a_2 & = & (1 + \frac{2}{1})/2 = \frac{3}{2} \\
a_3 & = & (\frac{3}{2} + \frac{2}{3/2})/2 = 1.41666... = \frac{17}{12} \\
a_4 & = & (\frac{17}{12} + \frac{2}{17/12})/2 = \frac{577}{408}
\end{array}$$

Now carry out this process in sexagesimal, beginning with $a_1 = 1;25$ using Babylonian arithmetic without rounding, to get the value 1;24,51,10.

Note: $\sqrt{2} \doteq 1;25 = 1.4166...$ was commonly used as a brief, rough and ready, approximation. When using sexagesimal numbering, a lot of information can be compressed into one place.

Solving Quadratics. The Babylonian method for solving quadratics is essentially based on completing the square. The method(s) are not as "clean" as the modern quadratic formula, because the Babylonians allowed only positive solutions. Thus equations always were set in a form for which there was a positive solution. Negative solutions (indeed negative numbers) would not be allowed until the 16th century CE.

The rhetorical method of writing a problem does not require variables. As such problems have a rather intuitive feel. Anyone could understand the problem, but without the proper tools, the solution would be impossibly difficult. No doubt this rendered a sense of the mystic to the mathematician. Consider this example

> *I added twice the side to the square; the result is 2,51,60.*
> *What is the side?*

In modern terms we have the simple quadratic $x^2 + 2x = 10300$. The student would then follow his "template" for quadratics. This template was the solution of a specific problem of the correct mathematical

type, all written rhetorically. Here is a typical example given in terms of modern variables. **Problem.** Solve $x(x + p) = q$.

Solution. Set $y = x + p$

Then we have the system

$$\begin{aligned} xy &= q \\ y - x &= p \end{aligned}$$

This gives

$$\begin{aligned} 4xy + (y - x)^2 &= p^2 + 4q \\ (y + x)^2 &= p^2 + 4q \\ x + y &= \sqrt{p^2 + 4q} \\ 2x + p &= \sqrt{p^2 + 4q} \\ x &= \frac{-p + \sqrt{p^2 + 4q}}{2} \end{aligned}$$

All three forms

$$\begin{aligned} x^2 + px &= q \\ x^2 &= px + q \\ x^2 + q &= px \end{aligned}$$

are solved similarly. The third is solved by equating it to the non-linear system, $x + y = p$, $xy = q$. The student's task would be to take the problem at hand and determine which of the forms was appropriate and then to solve it by a prescribed method. What we do not know is if the student was ever instructed in principles of solution, in this case completing the square. Or was mathematical training essentially static, with solution methods available for each and every problem that the practitioner would encounter.

It is striking that these methods date back 4,000 years!

Solving Cubics. The Babylonians even managed to solve cubic equations, though again only those having positive solutions. However, the form of the equation was restricted tightly. For example, solving $x^3 = a$

was accomplished using tables and interpolation. Mixed cubics

$$x^3 + x^2 = a$$

were also solved using tables and interpolation. The general cubic

$$ax^3 + bx^2 + cx = d$$

can be reduced to the normal form

$$y^3 + ey^2 = g$$

To do this one needs to solve a quadratic, which the Babylonians could do. But did the Babylonians know this reduction?

The Babylonians must have had extraordinary manipulative skills and as well a maturity and flexibility of algebraic skills.

Solving linear systems. The solution of linear systems were solved in a particularly clever way, reducing a problem of two variables to one variable in a sort of elimination process, vaguely reminiscent of Gaussian elimination. **Solve**

$$\frac{2}{3}x - \frac{1}{2}y = 500$$
$$x + y = 1800$$

Solution. Select $\tilde{x} = \tilde{y}$ such that

$$\tilde{x} + \tilde{y} = 2\tilde{x} = 1800$$

So, $\tilde{x} = 900$. Now make the model

$$x = \tilde{x} + d \qquad y = \tilde{y} - d$$

We get

$$\frac{2}{3}(900 + d) - \frac{1}{2}(900 - d) = 500$$
$$(\frac{2}{3} + \frac{1}{2})d + 1800/3 - 900/2 = 500$$
$$\frac{7}{6}d = 500 - 150$$
$$d = \frac{6(350)}{7}$$

So, $d = 300$ and thus

$$x = 1200 \qquad y = 600.$$

Plimpton 322 tablet
Yale Babylonian collection

5 Pythagorean Triples.

As we have seen there is solid evidence that the ancient Chinese were aware of the Pythagorean theorem, even though they may not have had anything near to a proof. The Babylonians, too, had such an awareness. Indeed, the evidence here is very much stronger, for an entire tablet of Pythagoreantriples has been discovered. The events surrounding them reads much like a modern detective story, with the sleuth being archaeologist Otto Neugebauer. We begin in about 1945 with the **Plimpton 322 tablet**, which is now the Babylonian collection at Yale University, and dates from about 1700 BCE. It appears to have the left section

broken away. Indeed, the presence of glue on the broken edge indicates that it was broken after excavation. What the tablet contains is fifteen rows of numbers, numbered from 1 to 15. Below we list a few of them in decimal form. The first column is descending numerically. The deciphering of what they mean is due mainly to Otto Neugebauer in about 1945.

$$
\begin{array}{cccc}
1.9834... & 119 & 169 & 1 \\
1.94915 & 3367 & 4825 & 2 \\
\vdots & & & \\
1.38716 & 56 & 106 & 15
\end{array}
$$

Interpreting Plimpton 322. To see what it means, we need a model right triangle. Write the Pythagorean triples, the edge b in the column thought to be severed from the tablet. Note that they are listed
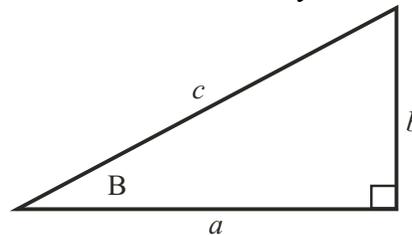
Right Triangle

decreasing cosecant.

b	$(c/b)^2$	a	c	
120	$(169/120)^2$	119	169	1
3456	$(4825/3456)^2$	3367	4825	2
$\vdots$				
90	$(106/90)^2$	56	106	15

$$
\csc^2 B = \left(\frac{c}{b}\right)^2
$$

A curious fact is that the tablet contains a few errors, no doubt transcription errors made so many centuries ago. How did the Babylonian mathematicians determine these triples? Why were they listed in this order? Assuming they knew the Pythagorean relation $a^2 + b^2 = c^2$, divide by b to get

$$
\left(\frac{a}{b}\right)^2 + 1 = \left(\frac{c}{b}\right)^2
$$

$$u^2 + 1 = v^2$$
$$(u - v)(u + v) = 1$$

Choose $u + v$ and find $u - v$ in the table of reciprocals.

**Example.** Take $u + v = 2;15$. Then $u - v = 0;26, 60$ Solve for u and v to get

$$u = 0;54, 10 \quad v = 1;20, 50.$$

Multiply by an appropriate integer to clear the fraction. We get $a = 65$, $c = 97$. So $b = 72$. This is line 5 of the table.

It is tempting to think that there must have been known general principles, nothing short of a theory, but all that has been discovered are tablets of specific numbers and worked problems.
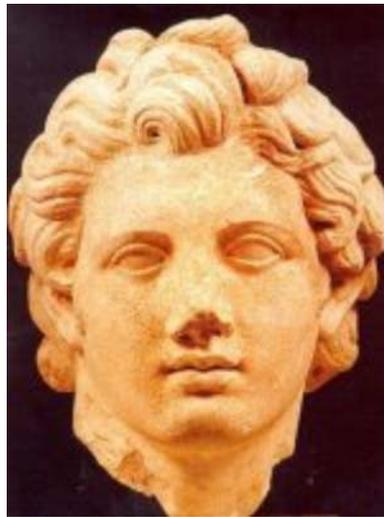
Bust of Alexander the Great
(356-323 BC) found at Thasos
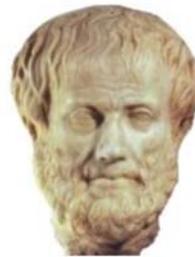
Aristotle (384-322 BC),
teacher of Alexander
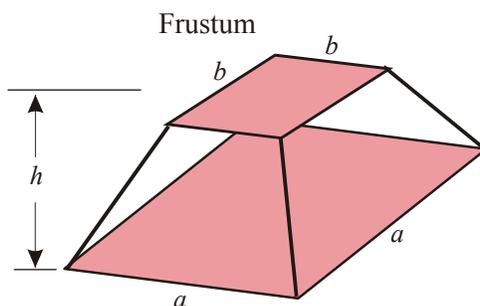
6   Babylonian Geometry

Circular Measurement.   We find that the Babylonians used $\pi = 3$ for practical computation. But, in 1936 at Susa (captured by Alexander the Great in 331 BCE), a number of tablets with significant geometric results were unearthed. One tablet compares the areas and the squares

of the sides of the regular polygons of three to seven sides. For example, there is the approximation

$$\frac{\text{perimeter hexagon}}{\text{circumference circumscribed circle}} = 0; 57, 36$$

This gives an effective $\pi \approx 3\frac{1}{8}$. (Not bad.)

Volumes. There are two forms for the volume of a frustum given

Frustum

$$V = (\frac{a+b}{2})^2 h$$

$$V = h\left((\frac{a+b}{2})^2 - \frac{1}{3}(\frac{a-b}{2})^2\right)$$

The second is correct, the first is not.

_____

There are many geometric problems in the cuneiform texts. For example, the Babylonians were aware that

- The altitude of an isosceles triangle bisects the base.
- An angle inscribed in a semicircle is a right angle. (Thales)

7 Summary of Babylonian Mathematics

That Babylonian mathematics may seem to be further advanced than that of Egypt may be due to the evidence available. So, even though

we see the development as being more general and somewhat broader in scope, there remain many similarities. For example, problems contain only specific cases. There seem to be no general formulations. The lack of notation is clearly detrimental in the handling of algebraic problems.

> There is an absence of clear cut distinctions between exact and approximate results.

Geometric considerations play a very secondary role in Babylonian algebra, even though geometric terminology may be used. Areas and lengths are freely added, something that would not be possible in Greek mathematics. Overall, the role of geometry is diminished in comparison with algebraic and numerical methods. Questions about solvability or insolvability are absent. The concept of "proof" is unclear and uncertain. Overall, there is no sense of abstraction. In sum, Babylonian mathematics, like that of the Egyptians, is mostly utilitarian — but apparently more advanced.

8   Exercises

1. Express the numbers 76, 234, 1265, and 87,432 in sexagesimal.

2. Compute the products

   (a) $1, 23 \times 2, 9$

   (b) $2, 4, 23 \times 3, 34$

3. A problem on one Babylonian tablets give the base and top of an isosceles trapezoid to be 50 and 40 respectively and the side length to be 30. Find the altitude and area. Can this be done without the Pythagorean theorem?

4. Solve the following system ála the Babylonian "false position" method. State clearly what steps you are taking.

$$
\begin{aligned}
2x + 3y &= 1600 \\
5x + 4y &= 2600
\end{aligned}
$$

(The solution is (200, 400).)

5. Generalize this Babylonian algorithm for solving linear systems to arbitrary linear systems in two variables?

6. Generalize this Babylonian algorithm for solving linear systems to arbitrary linear systems?

7. Modify the Babylonian root finding method (for $\sqrt{2}$) to find the square root of any number. Use your method to approximate $\sqrt{3}$. Begin with $x_0 = 1$.

8. Explain how to adapt the method of the mean to determine $\sqrt[3]{2}$.

9. Consider the table:

n	$n^3 + n^2$
1	2
2	12
3	36
4	80
5	150
6	252

Solve the following problems using this table and linear interpolation. Compare with the exact values. (You can obtain the exact solutions, for example, by using Maple: evalf(solve($x^3 + x^2 = a,\ x$)); Here a=the right side)

   (a) $x^3 + x^2 = 55$
   (b) $x^3 + x^2 = 257$

10. Show that the general cubic $ax^3 + bx^2 + cx = d$ can be reduced to the normal form $y^3 + ey^2 = g$.

11. Show how the perimeter identity is used to derive the approximation for π.

12. Write a lesson plan wherein you show students how to factor quadratics ála the Babylonian methods. You may use variables, but not general formulas.