The Origins of Mathematics[1]

The origins of mathematics accompanied the evolution of social systems. Many, many social needs require calculation and numbers. Conversely, the calculation of numbers enables more complex relations and interactions between peoples. Numbers and calculations with them require a well organized operational system. Such systems were perhaps the earliest models of complex rigorous systems. As we will see, not just one but several number systems come to us from antiquity. However, as interesting as the basic notions of counting may be, the origins of mathematics include more than just enumeration, counting, and arithmetic. Thus, also considered are other issues of mathematics to be considered.

Number provides a common link between societies and a basis for communication and trade. This chapter illustrates various entries into our ancient knowledge of how our number system began. Examples range from prehistory to contemporary. The ancient evidence is easy to accept, but the modern examples yield more conclusions. It is remarkable that even though mathematics achieved its first zenith twenty three hundred years ago, more than a hundred generations, there are peoples today that still count with their fingers or with stones, that have no real language for numbers, and moreover have not the general concept of number beyond specific examples. There is every reason to believe that in the future, near or distant, there will exist a far more credible theory for the origin of counting than there may ever for the development of geometric proof, the PythagoreanTheorem in particular.

1 The basis of computation

The human needs that inspired mankind's first efforts at mathematics, arithmetic in particular were

- counting,
- calculations,
- measurement

For example,

- The worth of a herdsman cannot be known unless some basics of counting are known.
- An inheritance cannot be distributed unless certain facts about division (fractions) are known.
- A temple cannot be built unless certain facts about triangles, squares, and volumes are known.

From practical needs such as these, mathematics was born. One view is that the core of early mathematics is based upon two simple questions.

- **How many?**
- **How much?**

This is the *cardinal number* viewpoint, which we will support throughout this chapter. The alternative *ordinal* basis is very attractive, with numerous supporting arguments.

1.1 Ordinals

Another view is that mathematics may have an even earlier basis on ordinals used perhaps for rituals in religious practices or simply the pecking order for eating the fresh game. It is a compelling argument that long before early man had need to count his sheep or cattle, his primitively organized social had simple rituals based on rank and priority. And not just who was first but also second, third, fourth, and so on, likely to most members of every group. Such basic questions are thus:

- **Who eats first, second, etc?**
- **What comes first, etc?**

There is ample observational evidence that socialized animals of diverse groups, say the baboon and red deer, have some rudimentary sense of order of precedence, called *dominance hierarchy*. The objects of the order or hierarchy are wide ranging also, for example, for grooming, for breeding, for eating, and the like.

Later on, the needs of counting and assigning numbers to sets of objects required more sophisticated calculation techniques, i.e. the beginnings of mathematics. There seems to be little direct path from ordinals to actual arithmetic. So, while ordinals may offer a plausible case for first origins, the evolution to cardinals was necessary before calculations became possible. Therefore, we will pursue the cardinal numbers viewpoint.

2 The Origin of Counting

Two possibilities for the origin of counting have been posited. One is that counting spontaneously arose throughout the world more or less independently from place to place, tribe to tribe. The other is that counting was invented just once and it spread throughout the world from that source. The latter view, maintained by Abraham Seidenberg, is based upon a remarkable number of similarities of number systems throughout the world. For example, that odd numbers are male and even numbers are female seems to be virtually universal. (Of course this distinction has been lost in modern times.) Seidenberg's anthropological studies further suggest that counting "was frequently the central feature of a rite, and that participants in ritual were numbered." Other mythical, ritualistic, and etymological evidence is also given.

What might be suggested from this unique and single origination of counting viewpoint is that the human mind was fairly uniform at this time (100,000 - 20,000 BCE), was ready for it, and moreover ready for it in about the same way. Tallying and ordering and counting originally seems to have served very similar purposes regardless of locale. On the other hand, these are exactly the same functions for which counting is primitively used today, giving credence to the alternative viewpoint that *all* uses were discovered early on and this has just not changed over dozens of millennia.

3 How Many

As society formed and organized, the need to express quantity emerged. Even at this early level, perhaps as early as 250,000 years ago, there must have begun a transition from sameness to similarity of numbers.

$$\text{one wolf} \rightarrow \text{one sheep}$$
$$\text{two dogs} \rightarrow \text{wolf} \rightarrow \text{two rabbits}$$
$$\text{five warriors} \rightarrow \text{five spears} \rightarrow \text{five fingers}$$

This abstraction of the concept of number was a major step toward modern mathematics.

From artifacts even more than 5,000 years old, notches on bones have been noted. Were these to count seasons, kills, children? We don't know. But the need to denote quantity must have been significant.

The English language, as others, has quantifier to indicate plurality

school of fish

pack of wolves

flock of pigeons

Examples of counting and computing. Other examples of counting and enumerations reveal how enumeration may have begun. They vary widely, illustrating a potential multitude of alternative origins for counting and arithmetic. This is significant for our brief study of these ideas: to consider, to understand, and to appreciate that our present arithmetic and counting was not a direct and continuous stream of advance. Indeed, no theory, no science, no exploration, and moreover, no human endeavor has ever so been.

1. The Indians of the Tamanaca on the Orinoco River.

Number	Word
0-4	designated words
5	"whole hand"
6	"one on the other hand"
⋮	
10	"both hands"
⋮	
20	"one Indian"
⋮	

Examples of peoples designating number-words as parts of the human anatomy are universal. (See Ifrah and Swetz.) Such enumeration may subconsciously become part abstract and part concrete. Eventually, the number-word emerges as a number without its anatomical association.

2. The Abipones, a tribe of Paraguayan Indians had a certain number sense, though they had little by way of number words or arithmetic. In the 18^{th} century a famine caused their migration for food. Their caravan was long and they were surrounded by their many, many dogs. When, in the evening they camped, they knew if even one was missing and went to lengths to find the missing animal. Similarly, they knew the size of a herd by knowing the space they occupied when they were lined up abreast. Their number words, one and two, were used in combination to form larger numbers, one-two for three, etc. Therefore, the sense they had for larger numbers, observable precise, was not verbal.

3. The Dammara tribe in Africa (19th century). In trading of tobacco sticks for sheep, the tribesman knew the equivalence:

   2 sticks = 1 sheep

   However, he was unable to cipher correctly the formula

   4 sticks = 2 sheep

   So, at the very early stage of counting, numerical equivalences such as

   two times two equals four

   are simply not obvious, and moreover just not used.

4. Certain Australian aboriginal tribes counted to two only, with any number larger than two called simply as much or many.

5. Other South American Indians on the tributaries of the Amazon, like the Abipones, were equally lacking in number words. They could count to six but had no words for three, four, five, or six. For example, four was expressed as two-two.

6. The bushmen of Africa could count to ten with just two words.

$$\text{ten} = 2 + 2 + 2 + 2 + 2$$

   For larger numbers the descriptive phrases became too long. The ease of number expression should not be underestimated in importance for the role of numerical facility.

7. The Vedda tribesman of Sri Lanka, when counting coconuts, would collect sticks and associate one stick with each coconut. When he added a new stick he would say, “That is one.” When asked how many coconuts he had, he would only point to his pile of sticks, saying, “That many.” The Vedda have no words to express quantities. For him counting was this one-to-one association.

8. There is an ancient account of trading that reveals another method of counting which supports the one-to-one correspondence concept. In the *History* by Herodotus (c. 484 - c. 424 BCE)[2] we find in Book IV this account of transactions between the Carthaginians and a tribe in Libya,

   > The Carthaginians also relate the following: — There is a country in Libya, and a nation, beyond the Pillars of Heracles, which they are wont to visit, where they no sooner arrive but forthwith they unlade their wares, and having disposed them after an orderly fashion along the

[2]The translation we reference here is that of George Rawlinson. Herodotus was a Greek historian that came from a Greek family of position of Halicarnassus, a Greek colony in Asia Minor. Long before the defeat of the Persians, Herodotus. living then under Persian tyranny, took up travel and reading. The published consequence, *History*, published sometime after 430 but before 425 reveals complete familiarity with the literature and an uncanny sense of observation and inquiry. While the *History* gained him fame, it also caused him political difficulties at his home. However, he was well received in Athens, where he enjoyed the company of the city’s literary elite. One lasting feature of Herodotus’ *History* is its narrative style, which allows for dialogue and even speeches by leading historical features, an aspect of historiography that persists to this day. One of his most frequently quoted statements is at once poetic and completely accurate. Said Herodotus, “Egypt is the gift of the Nile.”

> beach, leave them, and returning aboard their ships, raise a great smoke. The natives, when they see the smoke, come down to the shore, and, laying out to view so much gold as they think the worth of the wares, withdraw to a distance. The Carthaginans upon this come ashore and look. If they think the gold enough, they take it and go their way; but if it does not seem to them sufficient, they go aboard ship once more, and wait patiently. Then the others approach and add to their gold, till the Carthaginians are content. Neither part deals unfairly by the other; for they themselves never touch the gold till it comes up to the worth of their goods, nor do the natives every carry off the goods till the gold is taken away.

What we see here is a type of equivalence in a trade transaction. Do the natives regard the "ensemble" as individual items and add an amount of gold for each one, or is there a multiplication of the number of items by a unit cost?

Even earlier records

9. The earliest records of counting do not come from words but from physical evidence — scratches on sticks or stones or bones. For example, the oldest "mathematical artifact" currently known was discovered in the mountains between South Africa and Swaziland. It is a piece of baboon fibula with 29 notches, dated 35,000 CB. Old stone age peoples had devised a system of tallying by groups as early as 30,000 BCE. There is an example of the shinbone from a young wolf found in Czechoslovakia in 1937. It is about 7 inches long, and is engraved with 57 deeply cut notches, of about equal length, arranged in groups of 5. (Modern systems!!!) The oldest record of primes is possibly the **Ishango bone**. Currently at the Musee d'Histoire Naturelle in Brussels has been dated about 6500 CB. Having three rows of notches, and one of the columns has 11, 13, 17, and 19 notches, we may ask if primality was intended. Very probably not. What is more likely is that these notches formed a primitive calendar system.

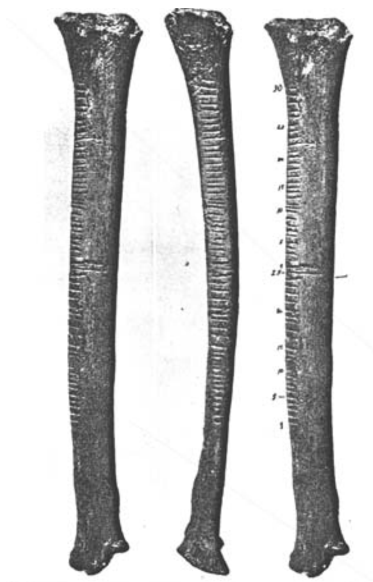
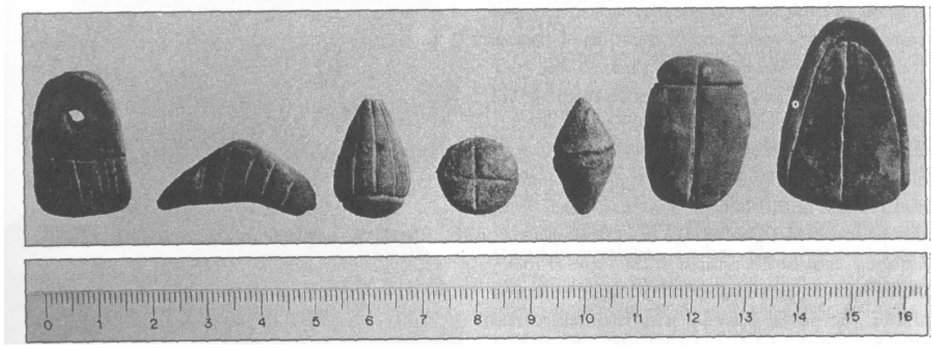


Diagram of the Ishango bone from the Brussels museum

See http://www.naturalsciences.be/expo/ishango/en/index.html for the Museum page on this remarkable object.

10. The first forms of fired clay tokens were used by Neolithic people to record products of farming (e.g. oil and grain) at sites near present day Syria and Iran, 8,000 BCE. Tokens are believed to have been instrumental in separating the ideas of number and written word. Such tokens were in use up until about 1,500 BCE.



Complex tokens from Susa, Iran: from left to right a parabola (garment?), triangle (metal?), ovoid (oil?), disk (sheep),

biconoid (honey?), rectangle (?), parabola (garment?). Courtesy Musee du Louvre, Department des Antiquities

Orientales.

11. There is other evidence dating from 8,500 B.C. on the shores of Lake Edward (in the Queen Elizabeth National Park in Uganda). An incised bone fossil contains groups of notches in three definite columns. Odd and unbalanced, it does not appear decorative. One set of is arranged in groups of

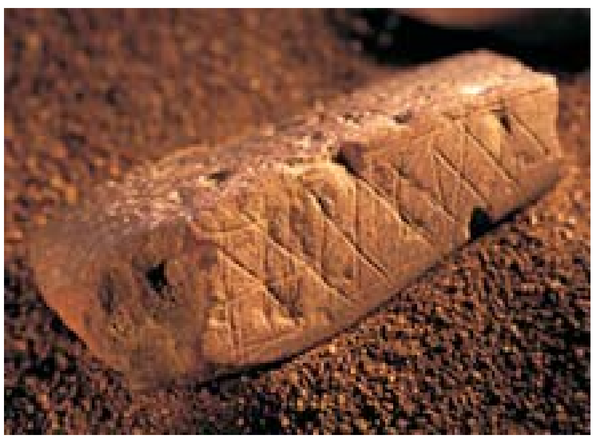
    11, 21, 19 and 9 notches. Another is arranged in groups of

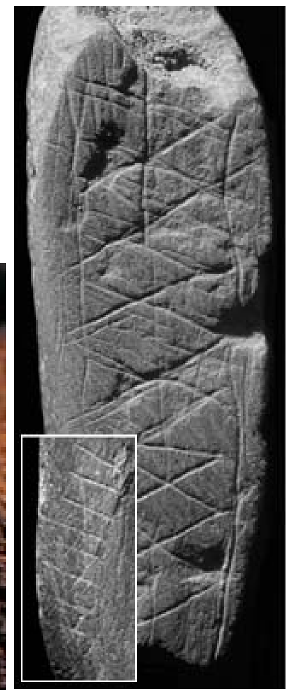
    3, 6, 4, 8, 10, 5, 5, 7 notches. Many have conjectured on the meaning of these groups. (Lunar months, doubling, halving, ...)

The ealiest record

In 2002, at the very southern tip of South Africa, scientists have recently uncovered in the Blombos cave some paleolithic art that dates back 70,000 years. That is more than 35,000 years older than any other 'stone age' art. Blombos Cave is located about 300 km east of Cape Town in a cliff directly above the Indian Ocean. About 90,000 years ago, when the sea was at its present level, (anatomically) modern Homo sapiens lived in the cave, living off the harvest of the sea and land - and it must have been a bountiful. Numerous artifacts have been found, including the oldest know bone tools, shellfish, hearths, and work areas. The age dates for the Middle Stone Age layers of between 90,000-100,000 years was obtained by the thermoluminescence, electron spin resonance and amino-acid racemization methods.

This discovery is remarkable for several reasons, the most notable being that it places 'modern behavior' at some earlier date than had been previously imagined. Another reason, and significant to mathematics, is that the two pieces of iron ore ochre rock found are decorated with geometric patterns. See the pictures below.





An interesting question, and this is completely open to conjecture, is whether this pattern had some mathematical or geometrical meaning beyond the natural appeal of the design. There are a number of websites on this site and the so-called Blombos stones. The pictures[3] above are from the website with URL http://www.accessexcellence.org/WN/SU/SU102001/caveart.html.

When we consider the Indians of North America, we will also observe more sophisticated but not dissimilar designs. There is

[3]See also, Blombos Cave and the Middle Stone Age of Africa, d'Errico, Francesco, Christopher Henshilwood and Peter Nillssen. 2001. An engraved bone fragement from c. 70,000-year-old Middle Stone Age levels at Blombos Cave, South Africa: Implications for the origin of symbolism and language. Antiquity 75:309-318.

ample reason to conjecture an earlier step to the development of mathematics than counting is rooted in early art. From art comes the evolution to patterns, and from patterns, separate unital objects such as criss-crosses become visible and ponderable. Perhaps withing the pattern of the Blombos stone, individual tokens were placed that represented events or other objects of daily life.

Below we see examples of cave paintings dating to 15,000 BCE that reveal early patterns. Such patterns occur even today in art. They form significant constructs in our mathematics, as well.

Cave paintings, c. 15,000 BC

Brazil

France

From from Richard Leakey's book *Origin's Reconsidered* we have the following quote about the cave paintings in the Lascaux cave in France.

> The most obvious interpretation of the scene in the Shaft is that it is connected with hunting magic, perhaps the reenactment of a hunting accident. But the most obvious explanation may not be the correct one, for three pairs of dots separate the rhinoceros from the rest of the scene. Simple in themselves, and perhaps without import, the dots are just one example of an element in Lascaux art, and in all cave art, that I have not yet mentioned. This is the profusion of nonrepresentational, geometric patterns. In addition to dots, there are grids and chevrons,

> curves and zigzags, and more. Many kinds of patterns are to be found, sometimes superimposed on animal images, sometimes separate from them. The coincidence of these geometric motifs with representational images is one of the most puzzling aspects of Upper Paleolithic art[4].

Conjecturing without bound we consider that such primitive patterns may have emerged as a visual analogue of sound patterns.

Some Etymology.

The words we use reveals to a great extent the origins of counting, grouping, and general record keeping. Some terms have at first glance unlikely origins, often coming from the process of counting, not the count itself. However, a consistency emerges when the ensemble of evidence is presented.

- The German word *schreiben* comes by way of Latin and Greek back to the word *gráphein*, originally meaning to scrape. Yet Germanic peoples had their own words for inscribing symbols, *ritzen* meaning "to scratch", and the Anglo-Saxon *writan*.
- The English word *tally* has a long heritage, It comes directly from the French verb *tailler*, "to cut". The medieval Latin root word is talare. The Romans used the word *talea*, a "cut twig."

  Note also the English word *write* comes from the Anglo-saxon *writan*, "to scratch".
- Even the word book takes its name from the word *Buche* or beach wood. The medieval manuscript call a *codex* comes from the Latin *caudex* or "log of wood."
- Our word *calculate* comes from the Latin *calculus*, pebble.
- The English *thrice*, just like the Latin *ter*, has the double meaning: three times, and many. There is a plausible connection between the Latin *tres*, three, and *trans*, beyond. The same can be said regarding the French

  trés, very, and *trois*, three.

[4] *Origins Reconsidered*, Leakey, Richard E and Roger Lewin. Doubleday (1992)

- Some primitive languages have words for every color but have no word for color. Others have all the number words but no word for number. The same is true for other words. Can you give and example? (e.g. animal, bird, tree fruit)
- English is very rich in native expressions for types of collections:

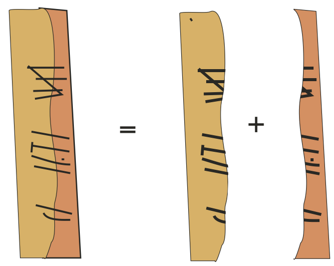
  flock, herd, set, lot, bunch, to name a few. Yet the words *collection* and *aggregate* are of foreign extraction. From Bertrand Russell we have the quote, "It must have required many ages to discover that a *brace* of pheasants and a *couple* of days were both instances of the number two." Today we have many terms to describe the idea of two:

  couple, set, team, twin, brace, etc.

Tallying

Tally sticks, a notched stick, have been used since the beginning of counting. Their use has been universal. Tallying is the most basic form of keeping a local record about the larger world. Before there was paper, and before paper became inexpensive, records were kept in a variety of ways. We will see the Babylonians use clay tablets and the Egyptians use stones. Both require a certain skill to create. Tally sticks can and were used by all peoples that have a ready piece of wood and something sharp. But it was not limited to "primitive" peoples. The acceptance of tally sticks as promissory notes or bills of exchange reached all levels of development in the British Exchequer tallies. (12^{th} century onwards.) It took an act of parliament in 1846 to abolish the practice.

An anecdote: The *double tally stick* was used by the Bank of England. If someone lent the Bank money, the amount was cut on a stick and the stick was then cut in half — with the grain of the wood. The piece retained by the Bank was called the **foil**, and the other half was called the **stock**. It was the receipt issued by the Bank. The holder of said became a "stockholder" and owned "bank stock". When the holder would return the stock was carefully checked against the foil; if they agreed, the owner would be paid the correct amount in kind or currency. A written certificate that was presented for remittance and checked against its security later became a "check". (See, David Burton, The History of Mathematics, An Introduction, McGraw-Hill.)



Double Tally Stick

Note how the tally marks match up exactly on the two split sticks. With forgery almost an impossibility, records kept using double tally sticks were very, very secure. It is no wonder they persisted so long.

Tallying on a bone or stick is both ancient and modern. A more ancient form of counting was done by means of knots tied in a cord — though counting is carried out to this day by knots or beads. Both objects and days were so tallied. From King Darius of Persia, we have this command given to the Ionians:

> *"The King took a leather thong and tying sixty knots in it called together the Ionian tyrants and spoke thus to them: "Untie every day one of the knots; if I do not return before the last day to which the knots will hold out, then leave your station and return to your several homes."*

Multicolored knotted cords, called *quipus* were also used by the Incas of Peru. The conquering Spaniards noted that in each village there were four *quipus* keepers, who maintained complex accounts and performed a function similar to today's city treasurer, historian, and secretary. The quipus, the only system used by the Incas, were usually 50 cm in length and with braids up to 40 cm extending. They could contain both alphabetic and numeric knot representations. For example, the number six is represented by a six-looped knot. It is of interest to note that quipus form an alternative to our traditional book/ledger format. They are essentially nonlinear, a structure that the Internet is allowing the rest of us to explore. Modern Internet expressions such as a threaded discussion is a "quipu-like" structure.

In the figure above, the pompom-like image is a quipu book, a rather dramatic departure from traditional codex format. It is supposed to contain the entire known history of the world, from creation to the 16th century, written on separate strips of paper. The picture below shows an Indian displaying a quipu. It has been suggested that in addition to their use for storage of numbers that they may also have been adapted to convey the sounds of the Inca language.

Knotted braids are used for record keeping even today, say by Bolivian herdsman. They were once used by the Chinese, as evidenced in a call by the philosopher Lao-tse, in the 5th century BCE, to return to the simple ways of doing things such as for example tying knots in cords to serve as writing.

History is replete with examples of knotted cords being used for record keeping, particularly numerical records. Even into the twentieth century Miller's knots were used by millers to record transactions with bakers. Pacific islanders kept track of wages owed by knotted reeds. Tibetan prayer-strings and the rosary are religious forms of number-strings. The heritage is old and ubiquitous.

- Systems of enumeration.

Primitive: notches, sticks, stones

Chinese: symbols for 1, 10, 100, 1,000

Egyptians: symbols for 1, 10, 100, 1,000, ... 1,0000,000.

Babylonians: two symbols only—*cuneiform*

Greeks: alphabetical denotations, plus special symbols, also a system similar to the Egyptian with special symbols

Roman: Roman numerals, I,V,X,L,C,D,M.

Arabs: Ten special symbols for numbers.

Modern: Ten special symbols for numbers.

- Methods of ciphering.

Devices: Abacus, counting boards.

Symbolic: Arithmetic.

Bases for numbering systems

- binary — early
- ternary — early

- quinary — early
- decimal
- vigesimal
- sexagesimal
- combinations of several

A study among American Indians showed that about one third used a decimal

scheme; one third

used a quinary/decimal scheme; fewer than a third used a binary scheme; and

about one fifth used a vigesimal system.

and a ternary scheme was used by only one percent.

HOW MUCH?

When counting or asking how many, we can limit discussions to whole positive

integers. When asking *how much*, integers no longer suffice. Examples:

Given 17 seedlings, how can they be planted in five rows?

Given 20 talons of gold, how can they be distribution to three persons?

Given 12 pounds of salt, how can it be divided into five equal containers?

When asking *how much* we are led directly to the need for **fractions**.

Another *how much* question is connected with *measurement*.

Where?

- Construction. To build granaries, or ovens to bake bread, or pyramids,

  or temples we need formulas for quantity, or area or volume.

- Planting. To divide arable plots we need formulas for plane area and

  those for seasons.

- Astronomy. To study the motions of stars we need angular and temporal

  measurement.

- Taxes and commerce. To properly assess taxes, we need ways to compute percentages

  (fractions).

To consider questions of *how much* we need more advanced numbers and

arithmetic; we also need concepts of **geometry**.

4 References

In addition to general references, the following may be useful.

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