History of Mathematics — Math 629

Second Group Homework: 30 January 2024

Not to be handed in, but should be discussed.

Some numerology of the Greeks

Do read Stillwell's exposition on how Euclid proved the Fundamental Theorem of Arithmetic, that every positive integer greater than 1 has an essentially unique factorization into powers of prime numbers.

1. Please do Stillwell's exercise 3.3.1, about the factors of a number of the form $2^{n-1}p$, where p is a prime.

2. Give the defining property of a perfect number. Prove that if $p = 2^n - 1$ is a prime number, then $2^{n-1}p$ is a perfect number. Write down four perfect numbers. How many perfect numbers are there currently known?

3. Let us call a positive number *ample* if the sum of its divisors exceeds the number itself. For example, $1 + 2 + 3 + 4 + 6 = 16 > 12$, so twelve is an ample number.

   Show that any number of the form $2^{n-1}(2^n - 1)$ is either perfect or ample.

   Give an ample number not of this form.