Honors Multivariate Calculus — Math 221H Section 201

Tenth Homework: Due in recitation: Thursday 2 November 2023

1. Evaluate the following improper integrals.

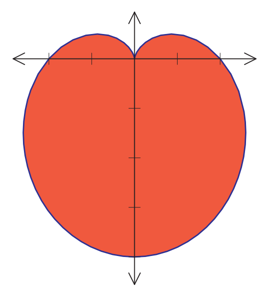
   (a) $\displaystyle\int_{-\infty}^{\infty} x^2 e^{-x^2}\,dx$ and (b) $\displaystyle\int_0^{\infty} \sqrt{x}e^{-x}\,dx\,.$

2. Electric charge is distributed over the disk $x^2+y^2 \le 2$ with charge density $\sigma(x,y) = 1+x^2+y^2$. Find the total charge on the disk.

3. A lamina (thin sheet of material) occupies the region inside the circle $x^2+y^2=2y$ but outside the unit circle $x^2+y^2=1$. Find its center of mass if the density at any point is inversely proportional to its distance to the origin.

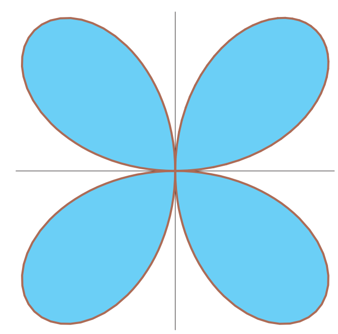
4. A lamina has shape that part of the disk $x^2+y^2 \le 4$ that lies in the first quadrant. Suppose that its mass density is proportional to the square of the distance to the origin, $\rho(x,y) = k(x^2+y^2)$. Find its center of mass.

5. Suppose that a lamina is in the shape of the rectangle $D = \{(x,y) \mid 0 \le x \le 2\,,\ 0 \le y \le 3\}$ with mass density $\rho(x,y) = \pi y$. Find its center of mass.



6. Find the three moments of inertial I_x, I_y, and I_0 of the lamina from each of the previous two problems.

7. Suppose that a lamina is in the shape of the cardiod $r = 2-2\sin(\theta)$ with constant mass density. Find its centroid and three moments of inertial I_x, I_y, and I_0.



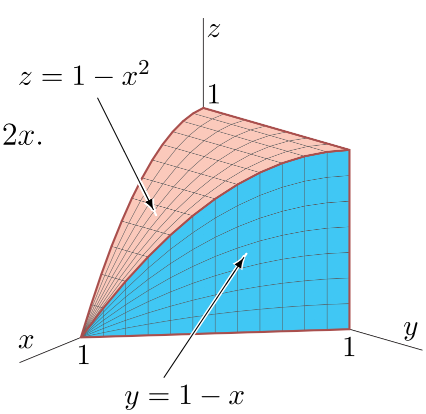
8. Repeat the previous exercise, but for the first loop (in the positive quadrant) of the four-leaved rose $r = 2\sin(2\theta)$. (Use symmetry to reduce your workload.)

9. Repeat the previous exercise, but for the full area enclosed by the four-leaved rose $r = 2\sin(2\theta)$. (Again, use symmetry.)

10. Evaluate the iterated integral $\displaystyle\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^x yz\,dy\,dz\,dx.$

11. Compute $\iiint_E xy\,dV$ where E is the tetrahedron with vertices $(0,0,0)$, $(3,0,0)$, $(0,2,0)$, $(0,0,1)$.

12. Compute $\iiint_E xy\,dV$ where E is the solid bounded by the parabolic cylinder $y = x^2$ and the planes $x = z$, $x = y$, and $z = 0$.



13. Express $\iiint_E f(x,y,z)\,dV$ as an iterated integral in six different ways, where E is the solid bounded by $z=0$, $x=0$, $y=2$, and $z = y-2x$.

14. The figure shows the region of integration for the integral

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x,y,z)\,dy\,dz\,dx\,.$$

Rewrite this as an equivalent interated integral in the five other ways.