Ninth Homework: Due in recitation: Thursday 26 October 2023

1. Evaluate the double integrals: (a) $\int_1^2 \int_0^1 (x+y)^{-2} dx\, dy$ (b)$\int_0^1 \int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}} dy\, dx$.

2. Find the volume of the solid under the hyperbolic paraboloid $z = x^2 - y^2$ and *above* the rectangle $[-1,1] \times [1,3]$ in the plane $z = 0$.

   What about the rectangle $[-1,1] \times [0,1]$?

3. In what way are the Theorems of Fubini and Clairaut similar?

   Suppose that $f(x,y)$ is continuous on $[a,b] \times [c,d]$. For (x,y) in this rectangle, define $g(x,y)$ to be the double integral $\int_a^x \int_c^y f(s,t) dt\, ds$. What is g_x? g_y? Show that $g_{xy} = g_{yx} = f(x,y)$.

4. Evaluate the double integrals.

   (a) $\iint_D (x^2 - 2xy) dA$, $D = \{(x,y) \mid 0 \le x \le 1\ \sqrt{x} \le y \le 2x\}$.

   (b) $\iint_D xy\, dA$, D is that part of the disk centered at the origin with radius 2, in the first quadrant.

5. Foe each of the following, sketch the region of integration and shange the order of integration.

   (a) $\int_1^2 \int_0^{\ln(x)} f(x,y)\, dy\, dx$. (b) $\int_0^1 \int_{\arctan(x)}^{\pi/4} f(x,y)\, dy\, dx$.

6. Evaluate the integrals by reversing the order of integration.

   (a) $\int_0^3 \int_{y^2}^9 y \cos(x^2), dx\, dy$. (b) $\int_0^1 \int_{\arcsin(y)}^{\pi/2} \cos x \sqrt{1 + \cos^2 x}\, dx\, dy$.

7. Find a Cartesian equation for the curve desribed by the given polar equations.

   (a) $r = \frac{5}{3 - 4\sin\theta}$. (b) $r^2 = \sin 2\theta$.

8. Find a polar equation for the curve represented by the given cartesian equations.

   (a) $x^2 = 4y$. (b) $x^2 - y^2 = 1$.

9. Use integration in polar coordinates to find the volume of the given solid.

   (a) Under the cone $z = \sqrt{x^2 + y^2}$ and above the annulus $4 \le x^2 + y^2 \le 25$ in the x, y-plane.

   (b) Bounded by the paraboloids $= 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.

10. Set up, but do not solve, an integral to find the volume of the solid that lies between the two paraboloids $z = 2x^2 + y^2 - 1$ and $z = 8 - x^2 - 2y^2$ and above the circle $x^2 + y^2 \le 3$.

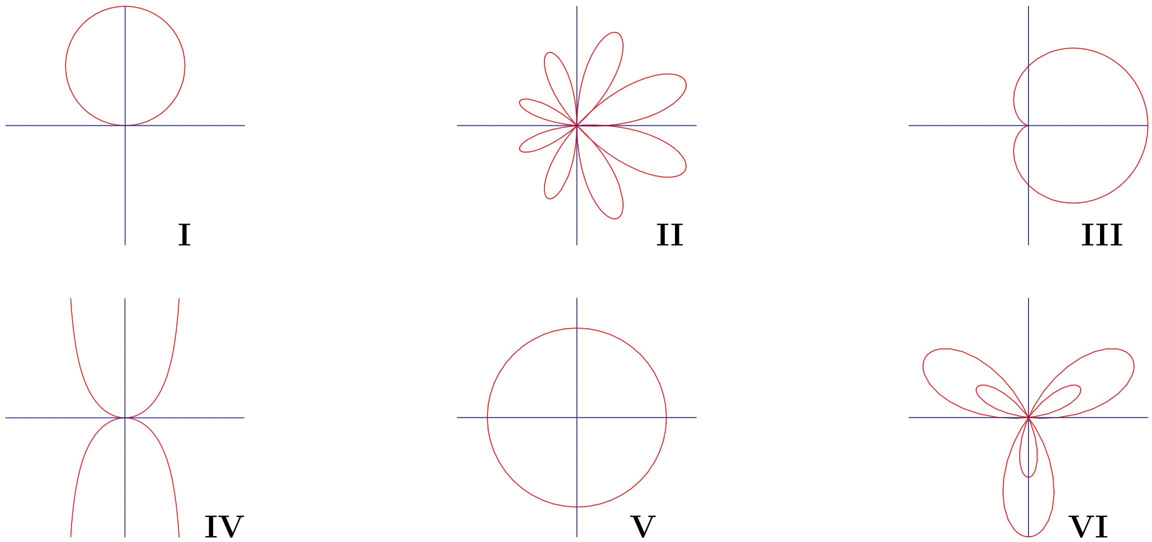
    Do this both in polar (r, θ) coordinates and in rectilinear (x, y) coordinates. This should be a double integral.

11. A cylindrical drill with radius r is used to drill a hole through the entre of a sphere with radius R, where $r < R$. Use integration in polar coordinates to find the volume of the ring-shaped solid that remains.

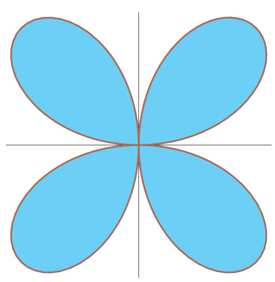
    Express this volume in terms of the height h of the ring.

12. Identify the polar curve with its polar equation.

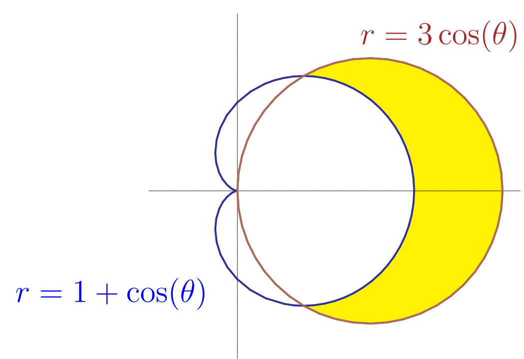
$(a)\quad r = \cos(\theta) - 1$	$(b)\quad r = 3/2$	$(c)\quad r = 1 + 3\sin(3\theta)$
$(d)\quad r = \cos(\theta) + 3\sin(4\theta)$	$(e)\quad r = \tan(\theta)$	$(f)\quad r = 2\sin(\theta)$



13. Use integration in polar coordinates to find the area enclosed by all four loops in the four-leaved rose $r = 2\sin(2\theta)$.



14. Use integration in polar coordinates to find the area of the shaded lune-shaped region which lies between the curve $r = 3\cos(\theta)$ and $r = 1 + \cos(\theta)$.



15. Use integration in polar coordinates to find the area enclosed by the lemniscate $r^2 = 4\cos(2\theta)$.