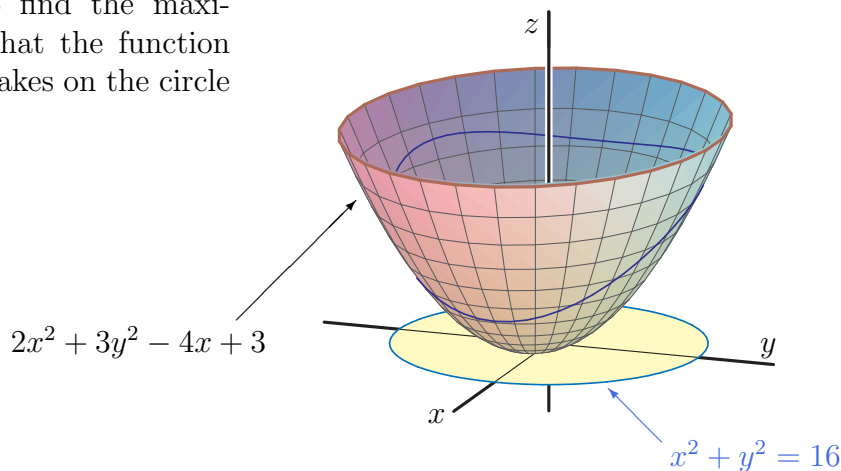


Homework about polynomial optimization using Lagrange multipliers

- Use the method of Lagrange multipliers to find the extreme values of the function x^3y on the ellipse $3x^2 + y^2 = 4$.
- Use the method of Lagrange multipliers to find the points on the surface $xy^2z^4 = 16$ that are closest to the origin.
- Use Lagrange multipliers to find the maximum and minimum values that the function $f(x, y) = 2x^2 + 3y^2 - 4x + 3$ takes on the circle $x^2 + y^2 = 16$.



- Let a, b, c be positive numbers. Use the method of Lagrange multipliers to find the volume of the largest box in the positive octant with three faces lying in the coordinate planes and one vertex on the plane $x/a + y/b + z/c = 1$.
- For each of the following, find the maximum of the given function $f(x, y)$ over the bounded set S . Use the methods of critical point analysis to find the maximum and minimum values over the interior of S and the method of Lagrange multipliers to find the maximum and minimum on the boundary of S .
 - $f(x, y) = x + y - xy$. $S = \{(x, y) \mid x^2 + y^2 \leq 9\}$.
 - $f(x, y) = 1 + \frac{x}{1+y^2}$. $S = \{(x, y) \mid x^2/4 + y^2/9 \leq 1\}$.
- Let $w = x_1x_2 \cdots x_n$.
 - Use Lagrange multipliers to maximize w subject to $x_1 + x_2 + \cdots + x_n = a$, where a is a fixed positive number and the x_i are positive, $x_1, \dots, x_n > 0$.
 - Use this to deduce the *Arithmetic Mean–Geometric Mean Inequality* for positive numbers a_1, \dots, a_n , which is

$$\sqrt[n]{a_1 a_2 \cdots a_n} \leq \frac{a_1 + a_2 + \cdots + a_n}{n}.$$