Honors Multivariate Calculus — Math 221H Section 201

Seventh Homework: Due in recitation: Thursday 5 Octobr 2023

Homework about polynomial optimization

1. Find and classify all the critical points in the domains of the following functions.

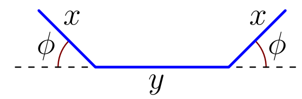
   (a) $2x^2 + 2xy + y^2 + 2x + 2y$. (b) $xy + \frac{2}{x} + \frac{3}{y}$. (c) $x^3 + y^3 - 6xy$.

2. Suppose that $f(x, y) = x^2 + y^2 + kxy$. Find and classify its critical points, and discuss how they change when k takes on different values.

3. Find the absolute maximum and minimum points of the function $f(x, y) = x^3 + 3y - 3xy$ over the region bounded by $y = x$, $y = 0$, and $x = 2$.

4. Find the absolute maximum and minimum points of the function $f(x, y) = x^2 + y^2 + x^2y + 4$ over the square $\{(x, y) \in \mathbb{R}^2 \mid -1 \le x, y \le 1\}$.

5. A length of metal sheet 2 meters wide is to be made into a trough by bending up equal strips along both sides. Find the width, x, of the strip of metal to be bent and the angle ϕ, so that the trough has maximum cross-sectional area.

6. Find the points on the surface $xy - z^2 + 1 = 0$ that are closest to the origin.

7. Find the volume of the largest box with edges parallel to the coordinate axes that may be inscribed in the ellipsoid $8x^2 + 9y^2 + 4z^2 = 72$.

8. Let a, b, c be positive numbers. Find the volume of the largest box in the positive octant with three faces lying in the coordinate planes and one vertex on the plane $x/a + y/b + z/c = 1$.

9. Consider the function $f(x, y) = x^3 - 3x^2y + y^3$. Show that $(0, 0)$ is the only critical point of f, and that the discriminant test is inconclusive for f.

   Determine the cross-sections of f obtained by considering the lines through the origin (e.g. set $y = kx$ for different values of k).

   What type of critical point for f is the point $(0, 0)$?

10. At which point (x, y) is the sum of the squares of the distance to the three points $(1, 4)$, $(5, 2)$, and $(3, 2)$ minimized?

    What is the distance from this point (x, y) to each of the three points?