Honors Multivariate Calculus Math 221H Section 201

Fifth Homework: Due in recitation: Thursday 21 September 2023

Homework about partial derivatives and tangent planes

1. Find the indicated partial derivatives.

   (a) $xyz = \sin(xy + yz + xz)$ $\dfrac{\partial z}{\partial x}$, $\dfrac{\partial z}{\partial y}$.

   (b) $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ $f_x(3, 0, 4)$, $f_y(0, 5, 12)$, $f_z(3, 4, 12)$.

   (c) $u = t^2x^3y^4$ u_t, u_x, u_y.

2. Find all second partial derivatives.

   (a) $f(x, y) = x^{\ln y}$ (b) $f(x, y) = \tan^2(3x + 2y)$ (c) $f(x, y, z) = xy^2z^3$

3. Suppose that f is a function of three variables with all higher order partial derivaties continuous. How many nth order (mixed) partial derivaties does f have, where $n = 1, 2, 3$? What is the general formula ?

4. Determine which of the following functions satisfies Laplace's equation, $u_{xx} + u_{yy} = 0$.

   (a) $u = x^3 + 3xy$. (b) $u = \sin x \cosh y + \cos x \sinh y$. (c) $u = e^{-x} \cos y - e^{-y} \cos x$.

5. Find an equation for the tangent plane to the given surface at the given point.

   (a) $z = x^2 - 2y^2 + 5y$ $(1, 2, 3)$

   (b) $z = \ln(3x + 2y)$ $(1, -1, 0)$

6. Use differentials to approximate the value of f at the given point.

   (a) $f(x, y, z) = x^2y^3z^4$ $(3.01, 2.02, 0.99)$

   (b) $f(x, y, z) = xy^2 \sin(\pi z)$ $(3.99, 2.98, 2.06)$

7. Use differentials to approximate the number $(\sqrt{63} + \sqrt[3]{9})^4$.

8. You observe your competitor's pile of sand, and want to know how much sand it holds. Viewing it remotely, you note that it is in the shape of a right circular cone 6 metres high and measure the angle of repose to be 60 degrees.

   (a) Given these measurements, how much sand does the pile hold?

   (b) If you reckon that your height measurement might be off by 10 centimetrers, and trust that you measured the angle of repose to within 2 degrees, what is the error in the volume? (use differentials and do not neglect that you are using degrees and not radians.