Honors Multivariate Calculus — Math 221H Section 201

Fourth Homework: Due in recitation: Thursday 14 September 2023

Homework about multivariate functions

All pictures are from the same vantage, looking in from the first (positive) orthant, and the axes all satisfy the right hand rule.

1. For each function below, determine its largest possible domain $D \subset \mathbb{R}^2$ and the range (image), which is the set $\{z \in \mathbb{R} \mid \exists (x,y) \in D \text{ with } z = f(x,y)\}$.

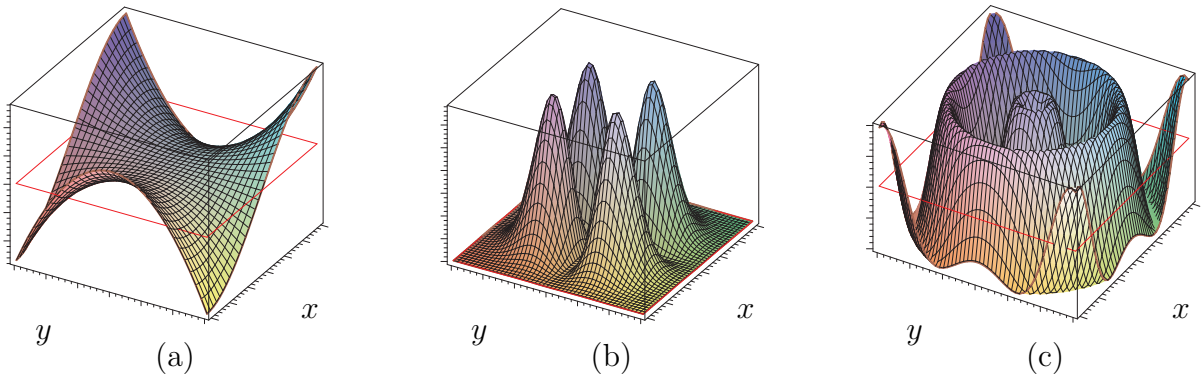
   (a) $f(x,y) = e^{x^2-y}$ (b) $f(x,y) = x^2 \ln(x+y-z)$ (c) $f(x,y) = \sqrt{100-4x^2-25y^2}$

2. For each function below, determine its largest possible domain $D \subset \mathbb{R}^2$ and sketch it.

   (a) $f(x,y) = \sqrt[4]{3x-2y}$ (b) $f(x,y) = \dfrac{x^2+y^2}{x^2-y^2}$ (c) $f(x,y) = \sqrt{1+x^2-y^2} + \ln(2-x^2-y^2)$

3. Match the function with its graph.

   (a) $f(x,y) = \sin\sqrt{x^2+y^2}$ (b) $f(x,y) = x^2y^2e^{-x^2-y^2}$ (c) $f(x,y) = x^3 - 3xy^2$



(a) (b) (c)

4. For the functions below, determine the largest set on which it is continuous.

   (a) $\sqrt{x+y} - \sqrt{x-y}$

   (b) $f(x,y) = \begin{cases} \frac{2x^2-y^2}{x^2+3y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

   (c) $f(x,y) = \begin{cases} \frac{y^2x}{x^2+3y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

5. Visual problem: Functions and their partial derivatives

Each triplet below shows the graphs of $f(x, y)$, $f_x(x, y)$, and $f_y(x, y)$ (in some order)x. For each identify which one is which. The red lines indicate $z = 0$ on the sides of the coordinate box.