Honors Multivariate Calculus — Math 221H Section 201

(This includes the quadrics Homework)

Third Homework: Due in recitation: Thursday 7 September 2023

Homework about quadric surfaces

1. Explore the interactive gallery of quadric surfaces. If at all possible, also please find a 3D visualization tool, such as Grapher (on an Apple Mac), or Geogebra, or Wolfram alpha, or some other software. Experiment with the software, using it to visualizing quadric surfaces and possibly their intersections with planes.

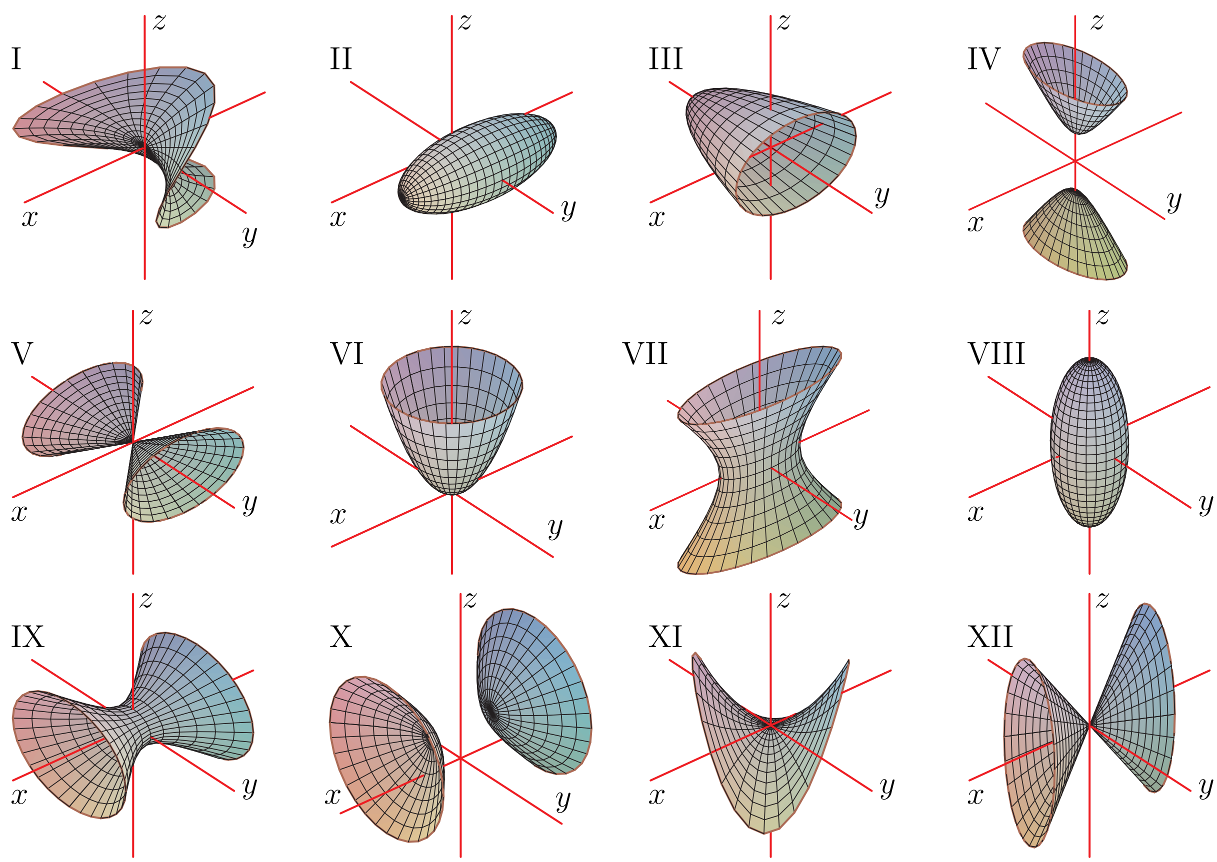
   Write a cogent paragraph or two describing *in your own words* what you did (briefly), how it went (also briefly), and what you learned.

   Consider the following quadratic equations in $\mathbb{R}^3$:

(a) $z = x^2 + y^2$	(b) $4x^2 + 25y^2 - 50y + 25z^2 = 0$	(c) $2y^2 + 4z^2 - 2x^2 = 1$
(d) $z = -xy$	(e) $x^2 - y^2 - z^2 + 2z = 2$	(f) $y^2 - 4z^2 = 4x$
(g) $y^2 = x^2 + 3z^2$	(h) $9z^2 + 4x^2 = 4y + 12$	(i) $25x^2 + 25y^2 + 4z^2 = 25$
(j) $2x^2 = z^2 + 8y^2$	(k) $z^2 - 8x^2 - 2y^2 = 1$	(l) $x^2 + 8y^2 - z^2 = 2$

2. For the equations (b), (d), (g), and (l), describe the cross sections for fixed x, fixed y, and fixed z.

3. Match each equation with one of the quadrics displayed below, and identify its type (ellipsoid, hyperbolic paraboloid, etc.)



4. Reduce the equation to one of the standard forms, classify the surface, and sketch it.
   (a) $4x^2 - 9y^2 + z^2 + 36 = 0$.
   (b) $y^2 - 2y = 2x^2 + z^2 + 2z$.
   (c) $4x - 8 = y^2 - 2z^2$.
   (d) $x^2 + y^2 - 4z^2 + 4x - 6y - 8z = 13$.

5. Find an equation for the surface consisting of all points P such that the distance from P to the y-axis is twice the distance of P to the xz-plane. What is this surface?

6. Find an equation for the surface consisting of all points P that are equidistant from the point $(0, 2, 1)$ and the plane $z = -1$. What is this surface?

7. Show that if the point (a, b, c) lies on the hyperbolic paraboloid $z = x^2 - y^2$, then the lines with the parametric equations

$$x = a + t \quad y = b + t \quad z = c + 2(a - b)t\,,$$

and

$$x = a + t \quad y = b - t \quad z = c + 2(a + b)t$$

both lie entirely on this paraboloid.

(This shows that the hyperbolic paraboloid is a *ruled surface*; it is generated by the motion og a straight line. In fact, this exercise shows that through each point on the hyperboic paraboloid, there are two such generating lines.)