Honors Multivariate Calculus Math 221H Section 201

YOUR NAME (-1 if you do not put your name here)

Second Homework: Monday 28 August 2023

Due in recitation: Thursday 31 August 2023

Do show your work for full credit.

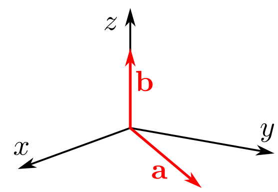
Homework about cross products

1. State whether each expression is meaningful. If not, explain why. If so, state whether it is a vector or a scalar.

   (a) $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})$ (b) $\mathbf{a}\times(\mathbf{b}\cdot\mathbf{c})$ (c) $(\mathbf{a}\cdot\mathbf{b})\times(\mathbf{c}\cdot\mathbf{d})$

   (d) $\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$ (e) $(\mathbf{a}\cdot\mathbf{b})\cdot\mathbf{c}$ (f) $(\mathbf{a}\times\mathbf{b})\cdot(\mathbf{c}\times\mathbf{d})$

2. The figure shows a vector $\mathbf{a}$ in the xy-plane and a vector $\mathbf{b}$ in the direction of $\mathbf{k}$. They have lengths $|\mathbf{a}|=4$ and $|\mathbf{b}|=2$.

   (a) Find $|\mathbf{a}\times\mathbf{b}|$

   (b) Use the right-hand rule to determine the signs $(+,-,0)$ of the components of $\mathbf{a}\times\mathbf{b}$.

3. Compute the cross products $\langle 1,2,-3\rangle\times\langle 4,-7,5\rangle$ and $(2\mathbf{i}+3\mathbf{j}-5\mathbf{k})\times(-7\mathbf{i}+11\mathbf{j}+13\mathbf{k})$.

4. Find a vector orthogonal to the plane of the three points, compute the area of the triangle they determine, and give an equation for the plane they lie on.

$$(0,2,4),\quad (-3,1,-5)\,,\quad (2,1,0)\,.$$

5. Prove the following formula for the vector triple product

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) \;=\; (\mathbf{a}\cdot\mathbf{c})\mathbf{b}-(\mathbf{a}\cdot\mathbf{b})\mathbf{c}\,.$$

6. Suppose that $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are vectors that are not coplanar. Define

$$\mathbf{k}_1 \;:=\; \frac{\mathbf{v}_2\times\mathbf{v}_3}{\mathbf{v}_1\cdot(\mathbf{v}_2\times\mathbf{v}_3)}\,,\qquad \mathbf{k}_2 \;:=\; \frac{\mathbf{v}_3\times\mathbf{v}_1}{\mathbf{v}_1\cdot(\mathbf{v}_2\times\mathbf{v}_3)}\,,\qquad\text{and}\qquad \mathbf{k}_3 \;:=\; \frac{\mathbf{v}_1\times\mathbf{v}_2}{\mathbf{v}_1\cdot(\mathbf{v}_2\times\mathbf{v}_3)}\,.$$

   (a) Show that if $i\neq j$, then $\mathbf{k}_i$ is orthogonal to $\mathbf{v}_j$.

   (b) Show that $\mathbf{k}_i\cdot\mathbf{v}_i=1$ for $i=1,2,3$.

   (c) Show that $\mathbf{k}_1\cdot(\mathbf{k}_2\times\mathbf{k}_3)=\dfrac{1}{\mathbf{v}_1\cdot(\mathbf{v}_2\times\mathbf{v}_3)}$.

Homework from section on equations for lines and planes.

7. Give the vector and parametric equations for the line passing through the point $(-3, 4, -5)$ and parallel to $\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$.

8. Show that the line through the points $(0, 1, 1)$ and $(1, -1, 6)$ is perpendicular to the line through the points $(-4, 2, 1)$ and $(-1, 6, 2)$.

9. Determine whether the two lines are parallel, skew, or intersecting. If they intersect, determine the point of intersection.

$$L_1\colon x = 1 + t\,,\ y = 2 - t\,,\ z = 1 + 3t \qquad L_2\colon x = 2 - s\,,\ y = 1 + 2s\,,\ z = 4 + s\,.$$

10. Find the equation for the plane through $(-1, 0, 1)$ that contains the line L_1 of the previous exercise.

11. Find the symmetric equation for the line of intersection of thetwo planes with the given equations, as well as the angle between the planes

$$x - 2y + z = 2 \quad \text{and} \quad x + 2y + z = 1\,.$$

12. Show that the lines $x = 2 + t$, $y = 2 - t$, $z = 2t$ and $x = -4 + 2s$, $y = 1 - s$, $z = 3 - s$ are skew, and determine the distance between them.

13. Find the parametric equation for the line through the point $(0, 1, 2)$ that is perpendicular to the line $x = 1 + t$, $y = 1 - t$, $z = 2t$, and which intersects that line.

14. Find an equation for the plane that contains the line of intersection of the two planes $x - z = 1$ and $y + 2z = 3$, and which is also perpendicular to the plane $x + y - 2z = 4$.