Honors Multivariate Calculus Math 221H Section 201

YOUR NAME (-1 if you do not put your name here)

First Homework: Monday 21 August 2023

Due in recitation: Thursday 24 August 2023

Homework from first section on vectors, 12.1

1. Draw a retangular box with faces parallel to the coordinate planes and whose opposite vertices are $(4, 3, 0)$ and $(1, 6, -4)$. Find the coordinates of the other six vertices and the length of a diagonal of the box.

2. Find the side lengths of the triangle with vertices $(3, -4, 1)$, $(5, -3, 0)$, and $(6, -7, 4)$. Is this triangle isosceles, right, both, or neither?

3. Suppose that $a^2 + b^2 + c^2 > 4d$. Show that $x^2 + y^2 + z^2 + ax + by + cz + d = 0$ is the equation representing a sphere, and find its center and radius.

4. Prove that the midpoint of the segment connecting the points (a, b, c) and (α, β, γ) is
$$\left(\frac{a+\alpha}{2}, \frac{b+\beta}{2}, \frac{c+\gamma}{2} \right).$$

5. Let X be the set of points P whose distance from $A = (-1, 5, 3)$ is twice its distance from $B = (6, 2, -2)$. That is, $X := \{ P \in \mathbb{R}^3 \ : \ |P - A| = 2|P - B| \}$. Show that X is a sphere and find its centre and radius.

6. Describe in words the region in $\mathbb{R}^3$ represented by the equation $x^2 + y^2 = 1$.

7. Describe in words the region in $\mathbb{R}^3$ represented by the equation $xyz = 0$.

Homework from second section on vectors, 12.2

8. Find the sum of the vectors $\langle 0, 3, 2 \rangle$ and $\langle 1, -2, 3 \rangle$, and illustrate this geometrically.

9. Find $|\mathbf{a}|$, $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$, $2\mathbf{a}$, and $3\mathbf{a} + 4\mathbf{b}$, where $\mathbf{a} = 6\mathbf{i} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + 7\mathbf{k}$.

10. Find the unit vector that has the same direction as $\langle 1, -4, 8 \rangle$.

11. Compute the dot product of $\langle -1, -2, -3 \rangle$ and $\langle 2, 8, -6 \rangle$.

12. Compute all six dot products between the vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.

13. Find the angle between $\langle 6, 0, 2 \rangle$ and $\langle 5, 3, -2 \rangle$. That is, give the exact expression and approximate the result to the nearest degree (you may need electronic machine asistance with the approximation).

14. For what values of c is the angle between the vectors $\langle 1, 2, 1 \rangle$ and $\langle 1, 0, c \rangle$ equal to $60°$?

15. Let $\mathbf{a} = \langle 3, -1 \rangle$ and $\mathbf{b} = \langle 2, 3 \rangle$. Compute $\text{proj}_{\mathbf{a}}\mathbf{b}$ and $\text{orth}_{\mathbf{a}}\mathbf{b} = \mathbf{b} - \text{proj}_{\mathbf{a}}\mathbf{b}$, and illustrate this by drawing all four vectors.

16. Find the angle between the diagonal of a cube and one of its edges.