Foundations of Mathematics — Math 300 Sections 902, 905

Thursday 12 November 2020 — Concept Quiz

Answers to Concept Quiz Sections 7.1-2

State of Relations. Let $\sim$ be the relation on the States of the United States in which for any two states S and T, we have $S \sim T$ if and only if S and T share a land border. For example Texas $\sim$ Oklahoma, but Texas $\not\sim$ California.

- Is Oklahoma $\sim$ New Mexico?

($\checkmark$) Yes

- Is Washington DC $\sim$ Virginia?

($\times$) No Washington DC is not a state.

- Is Missouri $\not\sim$ Tennessee?

($\times$) No They share a border.

Transitivity. Recall that a relation R on a set A is *transitive* if and only if for all $a, b, c \in A$, if aRb and bRc, then aRc.

- Let $\leq$ be the usual relation on the real numbers $\mathbb{R}$ of *less than or equal to*. Is $\leq$ transitive?

($\checkmark$) Yes This is one of the basic properties of $\leq$.

- Define $\sim$ on $\mathbb{R}$ by $a \sim b$ if $a + b$ is a rational number. Is $\sim$ transitive?

($\times$) No $2 + \pi \sim -\pi$ and $-\pi \sim \pi$, but $(2 + \pi) + \pi = 2 + 2\pi \notin \mathbb{Q}$, so that $2 + \pi \not\sim \pi$.

- Let m be a positive integer. Define the relation $\sim$ on $\mathbb{Z} \times \mathbb{Z}$ by $a \sim b$ if $a \equiv b \mod m$. Is $\sim$ transitive?

($\checkmark$) Yes We proved this earlier in the semester.

- Let $<$ be the relation defined for $r \in \mathbb{R}$ and $q \in \mathbb{Q}$ by $r < q$ if $q - r$ is positive. Is $<$ transitive?

($\times$) No This is not a relation on a set A, so it cannot be transitive.