Foundations of Mathematics — Math 300 Sections 902, 905

Thursday 24 September 2020 — Class worksheet

Definition. An integer a is *even* if there is an integer k such that $n = 2k$. An integer a is *odd* if there is an integer k such that $n = 2k+1$. (We assume the following result: Every integer n is either even or odd.)

Principle of Mathematical Induction. Let $P(n)$ be a statement for every positive integer n. Suppose that

1. $P(1)$ is true, and
2. for every positive integer k, if $P(k)$ is true, then $P(k+1)$ is true,

then $P(n)$ is true for all positive integers n.

1. Using the definitions, prove by cases that for every integer n, $n^2 - n + 41$ is odd.
2. What is the sum $2 + 5 + 8 + \cdots + (3n-1)$ equal to? Explore this, make a conjecture, and prove it.
3. Let $n \in \mathbb{N}$. What is the sum of the first n odd integers? Explore this, make a conjecture, and prove it.
4. Explore the divisibility by 3 of positive powers of 4. (E.g. $4^n \mod 3$.) Make a conjecture and prove it.