Foundations of Mathematics — Math 300 Sections 902, 905

Tuesday 22 September 2020 — Concept Quiz

Answers to Concept Quiz 3.3

1. What is a contradiction?

   A contradiction is a compound statement that is always false, whatever the truth values of its constituent statements.

   Which of the following statements are contradictions? P and Q are statements.

   - × $P \to (Q \vee \neg Q)$.
   - ✓ $P \wedge \neg P$.
   - ✓ $(Q \wedge P) \wedge (P \to \neg Q)$
   - × $P \to (Q \to \neg P)$.

2. Please briefly explain the logical basis for a proof by contradiction.

   We have the tautology: $(\neg X \to F) \to X$. That is, if the negation of X implies a falsehood, then X is true. In a proof of $P \to Q$ by contradiction, we assume $\neg(P \to Q) \equiv P \wedge \neg Q$, and then use this to deduce a contradiction. The previous tautology allows us to deduce that $P \to Q$. (There are other valid ways to explain this.)

3. What is the triangle inequality?

   For all real numbers x and y, we have $|x + y| \leq |x| + |y|$.

   This is in the book: Theorem 3.25(3) on page 137, and it does not involve triangles..