Foundations of Mathematics — Math 300 Sections 902, 905

Thursday 10 September 2020 — Class worksheet

Definition. Let a and b be integers with a nonzero. We say that a *divides* b if there exists an integer c such that $b = ac$. When this occurs, we write $a|b$.

Definition. Let n be a positive integer and a and b be integers. We say that *a is congruent to b modulo n* if n divides the difference $(a - b)$. When this occurs, we write $a \equiv b \mod n$.

1. Consider congruence modulo 5.
   (a) Choosing different pairs of integers a, b that are congruent modulo 5, what happens (e.g. with respect to congruence) when you add the same integer to each integer in a given pair?
   (b) The same question, but when you add two different integers which are themselves congruent modulo 5.
   (c) Try to formulate a conjecture about how congruence behaves when adding pairs of integers in this way.
   (d) What if you change 5 to any other positive integer?
2. Consider the conjecture we formulated about adding and congruence modulo 5
   (a) Construct a "know-show" table for a proof of this statement.
   (b) Write your proof in paragraph form.
3. Consider the following statement:

   $$\text{Let } n \in \mathbb{Z}. \text{ If } 5 \nmid (n^2 + 4), \text{ then } 5 \nmid (n - 1) \text{ and } 5 \nmid (n + 1).$$

   (a) Write its contrapositive
   (b) Construct a "know-show" table for a proof of this statement, in the form of a direct proof of the contrapositive.
   (c) Write your proof in paragraph form.