Foundations of Mathematics — Math 300 Sections 902, 905

Tuesday 8 September 2020 — Concept Quiz 3.1

Answers to Concept Quiz 3.1

1. **Parts of Mathematics**

   (a) In Mathematics, what is an **axiom**?
   An *axiom* is a mathematical statement that is accepted without proof.

   (b) Give an example of an axiom.
   In Euclidean Geometry: Any two distinct points determine a unique line.

   (c) In Mathematics, what is a **definition**?
   A *definition* is an agreement as to the meaning of a particular term.
   —William Fulton: If you make the correct definitions, the theorems prove themselves.

2. Indicate whether each of the following statements is true or false.

   (a) For every integers a and b, if $a|(b-1)$, then $a|(b^2-1)$.
   This is **True**. Note that $b^2-1=(b+1)(b-1)$, so if $a|(b-1)$, then $a|(b+1)(b-1)=b^2-1$.

   (b) For all integers a, b, and c with $a \neq 0$, if $a|(bc)$, then $a|b$ or $a|c$.
   This is **False**. Note that $4|36=6\cdot 6$, but $4 \not| 6$.

   (c) For all integers a, we have $a^3 \equiv a \mod 3$.
   This is **True**. Note that $a^3-a=a(a+1)(a-1)$. Observe that $a-1, a, a+1$ are three consecutive numbers, so that one is divisible by 3, and thus $3|a(a+1)(a-1)=a^3-a$ so $a^3 \cong a \mod 3$.

   You needn't have a proof, as that was not asked for.