Foundations of Mathematics — Math 300 Sections 902, 905

Tuesday 8 September 2020 — Class worksheet

Definition. Let a and b be integers with a nonzero. We say that a *divides* b if there exists an integer c such that $b = ac$. When this occurs, we write $a|b$.

Definition. Let n be a positive integer and a and b be integers. We say that *a is congruent to b modulo n* if n divides the difference $(a - b)$. When this occurs, we write $a \equiv b \mod n$.

1. Consider the following statement:

   For integers a, b, and c with $a \neq 0$, if $a|b$ and $a|c$, then $a|(b + c)$.

   (a) Explore how this may be true or not by trying some instances with a, b, c actual integers. Look for some aspect of this that you might be able to use in a proof.

   (b) Construct a "know-show" table for a proof of this statement.

   (c) Write your proof in paragraph form.

2. Consider congruence modulo 5.

   (a) Choosing different pairs of integers a, b that are congruent modulo 5, what happens (e.g. with respect to congruence) when you add the same integer to each integer in a given pair?

   (b) The same question, but when you add two different integers which are themselves congruent modulo 5.

   (c) Try to formulate a conjecture about how congruence behaves when adding pairs of integers in this way.

   (d) What if you change 5 to any other positive integer?

3. Consider the conjecture we formulated about adding and congruence modulo 5

   (a) Construct a "know-show" table for a proof of this statement.

   (b) Write your proof in paragraph form.