Foundations of Mathematics — Math 300 Sections 902, 905

Tuesday 1 September 2020 — Class worksheet

Note: $\mathbb{R}$ is the real numbers, $\mathbb{Q}$ is the rational numbers, $\mathbb{Z}$ is the integers, and $\mathbb{N}$ is the natural numbers (which begin with 0).

1. Write the converse and contrapositive of the following conditional statements

   (a) If it rains, then the grass is wet.

   (b) $\alpha^2 = 25$ if $\alpha = 5$.

   (c) The integer a is odd only if $3a$ is odd.

   (d) "Inattentive when bored"

2. Write a useful negation of each of the following statements. Do not leave a negation as the prefix of a statement. For example, the negation of "I will water my garden and pick basil" is "I will not water my garden or I will not pick basil".

   (a) You will walk or take the bus.

   (b) Knowledge is necessary for truth

   (c) Country and Western

   (d) If you wash the dishes or put away the laundry, you can have some chocolate.

3. Let a, b, and c be integers. Consider the following conditional statement:

   If a divides bc, then a divides b or a divides c.

   Which of the following statements have the same meaning as this conditional statement, and which are negations of this conditional statement"

   (a) If a divides b or a divides c, then a divides bc.

   (b) If a does not divide b or a does not divide c, then a does not divide bc.

   (c) a divides bc, a does not divide b, and a does not divide c.

   (d) If a does not divide b and a does not divide c, then a does not divide bc.

   (e) a does not divide bc or a divides b or a divides c.

4. Use the roster method to specify the elements in each of the following sets and then write a sentence in English describing the set.

   (a) $\{x \in \mathbb{R} \mid x^2 - 2x - 4 = 0\}$.

   (b) $\{n \in \mathbb{Z} \mid n^2 < 27\}$.

   (c) $\{n \in \mathbb{N} \mid n^2 < 27\}$.

   (d) $\{x \in \mathbb{Q} \mid x^2 - 2x - 4 = 0\}$.

5. Use set builder notation to specify the following sets.

   (a) The set of all natural numbers with square at least 15.

   (b) The set of all odd integers.

   (c) The set of all real numbers at most 10 whose square exceeds 3.

   (d) The set of positive rational numbers.