Foundations of Mathematics — Math 300 Sections 902, 905

Thursday 27 August 2020 — Class worksheet

1. Rewrite the following statements in the form "if P, then Q"
   (a) "One, if by land"
   (b) "Candor implies equality"
   (c) "Pepperoni only if pizza"
   (d) "Inattentive when bored"
   (e) "Slapstick is sufficient for comedy"
   (f) "Quiet is necessary for sleep"

2. Fill out the truth table for the expressions $P \wedge Q$, $\neg P$, $\neg Q$, $\neg(P \wedge Q)$, $\neg P \wedge \neg Q$, and $\neg P \vee \neg Q$:

P	Q	$P \wedge Q$	$\neg P$	$\neg Q$	$\neg(P \wedge Q)$	$\neg P \wedge \neg Q$	$\neg P \vee \neg Q$.

3. Rewrite "Friendship is necessary and sufficient for happiness" in four additional, equivalent ways.

4. Which of the following expressions are tautologies? Which are contradictions?
   (a) $(P \to Q) \vee (Q \to P)$
   (b) $(P \wedge Q) \vee (\neg P \wedge \neg Q)$
   (c) $P \to (Q \to P)$
   (d) $(P \wedge Q) \wedge (Q \to \neg P)$

5. Fill out a truth table (with $8 = 2^3$ rows) for the two expressions $(P \wedge Q) \vee (P \wedge R)$ and $P \wedge (Q \vee R)$. What do you observe?