Foundations of Mathematics — Math 300 Sections 902, 905

Tuesday 25 August 2020 — Class worksheet

Answers to Concept Quiz 2

1. Which of the following is a valid definition of an even integer? Check all that apply.

   N An integer n is even if it is not odd.

   Y An integer n is even if there is an integer a such that $n = 2a$.

   N An even integer n is an integer such that n^2 is even.

   N $\ldots, -4, -2, 0, 2, 4, \ldots$.

   N An integer n is even if there is a number r such that $n = 2r$.

Worksheet

Definition. An integer n is an *even integer* (or simply *even*) if there is an integer a such that $n = 2a$. An integer n is an *odd integer* (*odd*) if there is an integer a such that $n = 2a + 1$.

1. Consider the following statement: "If m is an even integer, then $m+1$ is an odd integer."

   Construct a know-show table for a proof of this statement.

   Write a proof of this statement in paragraph form.

2. Criticize (discuss its shortcomings) the following "proof" that if m and n are even, then $m+n$ is even:

   We know that $n = 2t$ and $m = 2t$, so $m+n = 2t + 2t = 4t$. Therefore $m+n$ is even.

   Write out a correct proof, first constructing a know-show table, and then writing it out in paragraph form.

3. Consider the following statement:

   "If m is an even integer and n is an integer, then mn is an even integer."

   Construct a know-show table for a proof of this statement.

   Write a proof of this statement in paragraph form.

4. Is the following statement true or false? Justify your conclusions.

   "If a, b, and c are integers, then $ab + ac$ is an even integer."