Math 151H Second Test
Sections 201 and 202

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Full credit is given only for complete and correct answers.
No aids allowed on the exam. Please write your answers in blue books.
Do persevere; partial credit will be given.
Point totals are in brackets next to each problem.

1. [10] Suppose that a function f is defined in a neighborhood of a point a. What does it mean for f to be differentiable at a? What is its derivative at a?

2. [10] Let $f(\xi) = \sqrt{\xi}$ for $\xi > 0$. Use the definition of derivative to compute $f'(a)$, where $a > 0$.

3. [15] Use *implicit differentiation* to compute γ' and γ'' if we have $\sqrt{x} + \sqrt{\gamma} = 5$. What are these derivatives when $x = 9$ and $\gamma = 4$?

4. [15] A runner runs around a circular track of radius 100 metres at a constant speed of 8 metres per second. The runner's coach is standing at a distance of 200 metres from the centre of the track. At what rate is the distance between them changing when the distance between them is 200 metres?

5. [10] Compute the trigonometric limit using methods from the course

$$\lim_{\omega \to \frac{\pi}{4}} \frac{\sin\omega - \cos\omega}{\cos 2\omega} \ .$$

6. [10] Let $f(v) := v^2 - 2$. Starting with the initial approximation of $x_0 = 1$ for the root $\sqrt{2}$ of f, use Newton's method twice to find the approximation x_3 to 6 decimal places.

7. [30] Compute derivatives with respect to the variable x of the following functions.

   (α) $\sqrt[5]{x\,\tan(x^2 e^x)}$ (β) $\left(x + \frac{1}{x^2}\right)^{\sqrt{7}}$ (γ) $\csc(1 + x + x^3)$

   (δ) $\sin(x\,\cos(x))$ (ϵ) $\sec(e^{\tan x})$ (ζ) $x^e + e^x$

   (η) $\dfrac{\cos(1+x)}{\cot(1-x)}$ (θ) $\dfrac{3x^2 - x + 1}{e + \sqrt{\sin(e^2)}}$

Ω. [5] (Literacy Bonus) In the 1980's, underfunding of California's schools led to a 60% decline in standardized test scores. Vigorous reforms led to a 70% increase in test scores during the 1990's. What was the net percentage change in test scores during these two decadees?