Math 151H
Sections 201 and 202
First Test

23 September 2008
Instructor: F. Sottile

Full credit is given only for complete and correct answers.
No aids allowed on the exam. Please write your answers in blue books.
Do persevere; partial credit will be given, and you are all good students.
Point totals are in brackets next to each problem.

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1. (a) [10] Suppose that f is a function and l, a are real numbers. Give the precise ϵ–δ definition of *limit*. That is, give the definition of: "The function f approaches the limit l near a".

   (b) [25] Using this definition of limit, prove that $\lim_{x\to 4}\left(\frac{x}{3}+1\right) = \frac{7}{3}$.

2. [10] Let $\mathbf{v} := \langle -7, 24\rangle$ and $\mathbf{w} := \langle 3, 4\rangle$.

   a) Compute $|\mathbf{v}|$. b) Compute the scalar and vector projections of $\mathbf{v}$ along $\mathbf{w}$

3. [15] State the limit laws, besides $\lim_{x\to a} c = c$ and $\lim_{x\to a} x = a$, for $a, c \in \mathbb{R}$.

   Given that $\lim_{x\to a} f(x) = 2$, $\lim_{x\to a} g(x) = 5$, and $\lim_{x\to a} h(x) = 11$, find the following limits that exist.

   a) $\lim_{x\to a}[f(x) + h(x)]$. b) $\lim_{x\to a} \dfrac{f(x)}{h(x) - g(x)}$.

4. [5] Evaluate $2 - \cfrac{1}{3 - \cfrac{1}{2 - \cfrac{3}{2}}}$.

5. [5] Define $\csc(x)$ in terms of the unit circle. What is $\csc(\pi/6)$?

6. [10] Show that $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y$.

7. [10] Give two radically different definitions for the number π.

8. [10] Find all real numbers x such that $x^4 - 1 > 0$. Display your answer on a number line.