Second Homework Assignment for Math 151H
Due September 11

Factor the following polynomials as much as you can, using integer coefficients.

1. $x^2 - 1$.
2. $x^3 - 1$.
3. $x^2 - a^2$.
4. $x^3 - a^3$.
5. $x^4 - a^4$.
6. $x^5 - a^5$.
7. $x^6 - a^6$.
8. $x^7 - a^7$.
9. $x^8 - a^8$.

10. Find a formula for
$$\sum_{i=1}^{n}(2i-1) \;=\; 1+3+5+\cdots+(2n-1).$$
The coefficient $\binom{n}{i} := \frac{n!}{i!(n-i)!}$ of $a^{n-i}b^i$ in the expansion $(a+b)^n$ is called a *binomial coefficient*. That is, $(a+b)^n$ equals
$$\tbinom{n}{0}a^n + \tbinom{n}{1}a^{n-1}b + \cdots + \tbinom{n}{n-1}ab^{n-1} + \tbinom{n}{n}b^n.$$
11. Show that $\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n$.
12. When $n = 7$, we noticed that all except the first and last binomial coefficient was divisible by 7. Can you explain this using the formula?
13. What about binomial coefficients $\binom{p}{i}$ when p is a prime number?
14. Use induction to show that
$$1 + r + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}.$$
15. What is the simplest function? Why?
16. What is the second-simplest type of function? Explain.
17. If f and g are functions, what is $f + g$?
18. If f and g are functions, what is $f \cdot g$?
19. What is a polynomial function?
20. What is the domain of a polynomial function?
21. What is a rational function?
22. What is the domian of a rational function?
23. Does there exist a rational function with non-constant denominator and domain all of $\mathbb{R}$?
24. What is π?
25. What is π? (This is not a typo. There are two answers.)
26. Define $\sin x$ in terms of the unit circle.
27. Define $\cos x$ in terms of the unit circle.
28. What is $\tan x$?
29. What is $\sec x$?
30. What is $\cot x$?
31. What is $\csc x$?
32. Show that $\sin(-x) = -\sin x$.
33. Show that $\cos(-x) = \cos x$.
34. Show that $\sin(y+z) = \sin y \cos z + \cos y \sin z$.
35. Derive a formula for $\tan(2\omega)$.
36. Show that $\sin(\frac{\pi}{2} - \alpha) = \cos\alpha$.
37. Give the numerical values of $\sin\frac{\pi}{3}$, $\cos\frac{\pi}{3}$, $\tan\frac{\pi}{3}$, $\sec\frac{\pi}{3}$, $\cot\frac{\pi}{3}$, and $\csc\frac{\pi}{3}$ as radicals.
38. Give the numerical values of $\sin 0$, $\cos 0$, $\tan 0$, $\sec 0$, $\cot 0$, and $\csc 0$ as radicals.
39. Give the numerical values of $\sin\frac{\pi}{2}$, $\cos\frac{\pi}{2}$, $\tan\frac{\pi}{2}$, $\sec\frac{\pi}{2}$, $\cot\frac{\pi}{2}$, and $\csc\frac{\pi}{2}$ as radicals.
40. Give the numerical values of $\sin(-\frac{\pi}{6})$, $\cos(-\frac{\pi}{6})$, $\tan(-\frac{\pi}{6})$, $\sec(-\frac{\pi}{6})$, $\cot(-\frac{\pi}{6})$, and $\csc(-\frac{\pi}{6})$ as radicals.
41. Show that $\dfrac{\sec a - 1}{\sec a + 1} + \dfrac{\cos a - 1}{\cos a + 1} = 0$.
42. Show that $1 + \cot^2(\pi/2 - x) = \dfrac{1}{\sin^2(\pi/2 - x)}$.
43. Show that $\dfrac{\sin\beta}{\csc\beta} + \dfrac{\cos\beta}{\sec\beta} = 1$.
44. Show that $\sec^4\theta - \sec^2\theta = \dfrac{1}{\cot^4\theta} + \dfrac{1}{\cot^2\theta}$.
45. Show that $\tan 3\beta = \dfrac{3\tan\beta - \tan^3\beta}{1 - 3\tan^2\beta}$.
46. Show that $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y$.
47. Show that $\cot(x/2) = \dfrac{1 + \cos x}{\sin x}$.
48. Show that $\csc y \sec y = 2\csc 2y$.