Math 151H

Sections 201 and 202

First Test

21 September 2006

Instructor: F. Sottile

Full credit is given only for complete and correct answers.
No aids allowed on the exam. Please write your answers in blue books.
Do persevere; partial credit will be given, and you are all good students.
Point totals are in brackets next to each problem.

1. (a) [10] Suppose that f is a function and l, a are real numbers. Give the precise ϵ-δ definition of *limit*. That is, give the definition of: "The function f approaches the limit l near a".

   (b) [25] Using this definition of limit, prove that $\lim\limits_{x \to 5} \sqrt{x} = \sqrt{5}$.

2. [10] Let $\mathbf{v}$ be the vector $\langle -12, 5 \rangle$.

   a) Compute $|\mathbf{v}|$.

   b) Compute the dot product $\mathbf{v} \cdot \langle 3, 5 \rangle$

3. [15] Given that $\lim\limits_{x \to a} f(x) = -3$, $\lim\limits_{x \to a} g(x) = 1$, and $\lim\limits_{x \to a} h(x) = 8$, find the following limits that exist.

   a) $\lim\limits_{x \to a} [f(x) + h(x)]$

   b) $\lim\limits_{x \to a} \dfrac{f(x)}{1 - g(x)}$

   c) $\lim\limits_{x \to a} \left(f(x)h(x) - \dfrac{g(x)}{(h(x))^2 - 20 f(x)} \right)$

4. [10] Suppose that A has been defined as a 4×6 matrix. Write a MATLAB command to create a column vector $\mathbf{x}$ from the second and third rows of A.

5. [15] Evaluate the following limit.

   $$\lim_{x \to 0} \frac{x}{\sqrt{2} - \sqrt{2 + x}}.$$

6. [15] Recall that if f and g are functions, then $f \circ g(x)$ is $f(g(x))$. Suppose that $f(x) = \sqrt{x - 1}$ and $g(x) = x^2$. Give formulas and the largest domain of definition for $f \circ f$, $f \circ g$, $g \circ f$, and $g \circ g$.