Math 171, Section 502
Third Test

26 April 2005
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Full credit is given only for complete and correct answers.
No aids allowed on the exam. Please write your answers in blue books.
Do not simplify your answers to the derivatives.
Point totals are in brackets next to each problem. 100 points total.

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1. [10] State Lagrange's Mean Value Theorem.
2. [10] Compute the derivative of $\sin^{-1}\sqrt{x}$.
3. Use L'Hospital's Rule to evaluate the following indeterminate forms.

   (a) [10] $\lim_{x\to 0}\dfrac{\tan^{-1}(2x)}{3x}$. (b) [10] $\lim_{x\to 0^+}(3x)^x$.

4. Let $f(x) = x^4 - 6x^2 + 4$

   (a) [10] Find all local maximum and minimum values of f.

   (b) [10] Find the intervals of concavity and any inflection points.

5. [20] A box with a square base and open top must have volume of 32 cubic inches is to be made from a sheet of palladium. Find the dimensions of the box that minimizes the amount of material used.
6. Recall that the Fundamental Theorem of Calculus states that $\int_a^b f(x)dx = F(b) - F(a)$, where $F(x)$ is an anti derivative of the continuous function f.

   Evaluate the following integrals.

   (a) [10] $\displaystyle\int_1^2 (x^2 + \tfrac{1}{x})\ dx$ (b) [10] $\displaystyle\int_0^{\pi/4} \sec x \tan x\ dx$

7. **Extra Credit** [10] For each of parts (a) and (b), you will need the formula

   $$(*) \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

   valid for any positive integer n.

   (a) Show that $\int_0^1 (x^2+1)\ dx = \frac{4}{3}$ by using $(*)$ and taking the area to be the limit of the sum of the areas of rectangles approximating the region.

   (b) Prove the formula $(*)$ by using mathematical induction.