Math 171, Section 502
Second Test

Full credit is given only for complete and correct answers.

No aids allowed on the exam. Please write your answers in blue books.

Do not simplify your answers to the derivatives.

Do persevere; partial credit will be given, and you are all good students.

Point totals are in brackets next to each problem. 100 points total.

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1. [10] Give the precise definition for the derivative of a function $f(x)$ at a number $a \in \mathbb{R}$.

2. [10] Compute the following trigonometric limit. Do not use L'Hospital's rule.

$$\lim_{\theta \to 0} \frac{1 - \cos \theta}{\sin^2 \theta}$$

3. Compute the derivative y' of the following functions.

   (a) [10] $y = \tan(e^x + x^2)$

   (b) [10] $y = \dfrac{x^2 e^x}{\cos x^2}$

   (c) [10] $y = \sin^2(\ln x)$

   (d) [10] $y = x^x$

4. [15] Find the velocity and acceleration of a particle with position $\mathbf{r}(t)$ given by

$$\mathbf{r}(t) \ := \ \langle t^4 - 7t^2, \ t^2 + 2t \rangle .$$

   Is the particle ever at rest?

5. [10] Compute y' where y is a function of x defined implicitly by

$$\sqrt{xy} \ = \ 2x - \sqrt{y} .$$

6. [15] A spherical snowball melts in such a way that its volume is decreasing at a rate of 10 cm^3 per second. How fast is its diameter decreasing when the diameter is 10 cm?

   Extra Credit

7. [optional bonus 10] Let $f(x) = x^2 \cos \frac{1}{x}$. What is $f'(0)$? Justify your answer.