Math 171, Section 502
First Test

16 February 2005
Instructor: F. Sottile

Full credit is given only for complete and correct answers.
No aids allowed on the exam. Please write your answers in blue books.
Do persevere; partial credit will be given, and you are all good students.
Point totals are in Brackets next to each problem.

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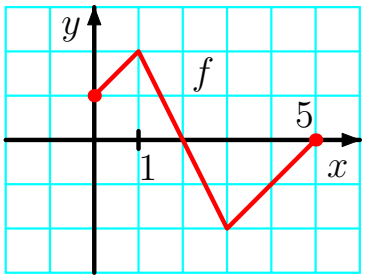
1. (a) [10] Give the precise ϵ-δ definition of *limit*, that is $\lim_{x \to a} f(x) = l$ means:

   (b) [10] Using the definition of limit, prove that $\lim_{x \to 2} 3x + 1 = 7$.

2. [15] The graph of a function f is shown to the right.
   What is its domain?
   On different grids with labeled axes,
   draw graphs of the following functions
   a) $y = f(x + 2)$ b) $y = f(x) + 2$
   c) $y = -2f(x) + 2$



3. [20] Let $\mathbf{v}$ be the vector $\langle -3, 5 \rangle$.
   a) Compute $|\mathbf{v}|$. b) Give a vector perpendicular to $\mathbf{v}$.
   c) Compute the dot product $\mathbf{v} \cdot \langle 12, 7 \rangle$
   d) Give a unit vector in the direction of $\mathbf{v}$.

4. [15] Evaluate the following limit, justifying each step using the limit laws.
   $$\lim_{x \to 3} (x^3 + 2x^2 + 6).$$

5. [20] Evaluate the following limit $\lim_{t \to 0} \dfrac{\sqrt{2 - t} - \sqrt{2}}{t}$. There is no need to justify your steps.

6. [10] Recall that a function f is *continuous at* $x = a$ if $\lim_{x \to a} f(x) = f(a)$.

   Brief essay. Write a short paragraph explaining why the following statement is true. For example, if it uses a Theorem from the course, give a rough statement of the Theorem or its name (e.g. "Intermediate value Theorem"), and how it applies.

   A polynomial function $P(x)$ is continuous at every real number a.