

# Hyperbolic Soccer Ball

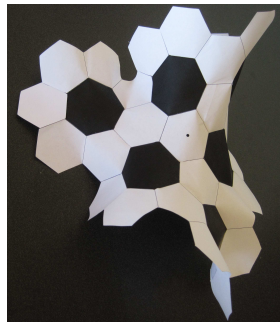
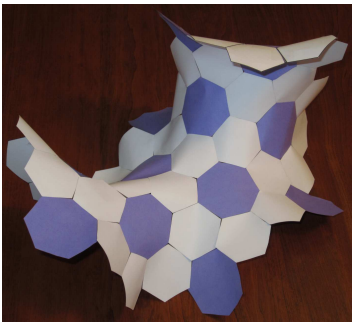
## Frank Sottile

For more information, see [www.math.tamu.edu/~sottile/research/stories/hyperbolic\\_football/](http://www.math.tamu.edu/~sottile/research/stories/hyperbolic_football/) or google "sottile hyperbolic football".

Constructing and studying the hyperbolic soccer ball is an enjoyable activity resulting in a beautiful model which can be used to investigate the hyperbolic plane and non-Euclidean geometry. With suitable modifications, this has been used in many settings, including a **middle school math circle**, in a teacher-prep course on non-Euclidean geometry, at university math clubs, and even with graduate students and faculty (at the University of Ilorin in Ilorin, Nigeria).



Templates for constructing hyperbolic soccer balls are available as .pdf files from the above web page. You need to print out enough for your circle (**including enough for the enthusiastic students who will want to make a giant model.**) If working with a younger crowd, or under a time constraint, you may need to cut out some or all of the templates prior to your circle for the students to assemble. In any event, you should certainly build a couple of models to get an idea of how this activity may go, and to decorate your office. Here are some models.



Use your models to investigate non-Euclidean geometry. Here is a selection of some activities:

**Draw a line.** (See reverse side)

**Diverging parallels.** (See reverse side)

**Parallel Postulate.** Given parallel lines as in the previous activity, extend a new perpendicular segment from one ( $\ell$ ) meeting the second at point  $P$ . The line perpendicular to the segment at  $P$  is parallel to  $\ell$ , giving two parallels to  $\ell$  through  $P (\notin \ell)$ . Thus Euclid's parallel postulate, or rather the equivalent Playfair's axiom (*At most one line can be drawn through any point not on a given line parallel to the given line in a plane*) does not hold. (This also constructs a Lambert quadrilateral (*three right angles*), but that is another story.)



**Triangle.** (See reverse side)

**Curvature.** The model is flat, except at its vertices, each of which contributes  $-60/7^\circ \approx -8.5^\circ$  of curvature in a  $\delta$ -function. (This is the *excess* at the vertex, the difference of  $360^\circ$  and the sum of the incident angles.) This gives the deviation from Euclidean geometry. For your triangle check that

$$\text{angle sum} + 8.5^\circ \cdot \# \text{ vertices inside triangle} = 180^\circ.$$

For the triangle at right, we have  $28 + 43 + 40 + 8 \cdot 8.5 = 179.0$ . Have your students check this relation, or better have them compare each other's angle sums and make it a very fascinating guided discovery activity.

