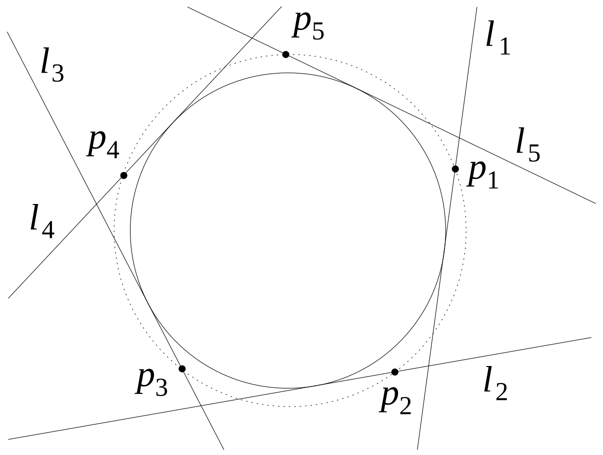
Conics from the configuration of Figure 1 in §4 of "Enumerative geometry for real varieties".

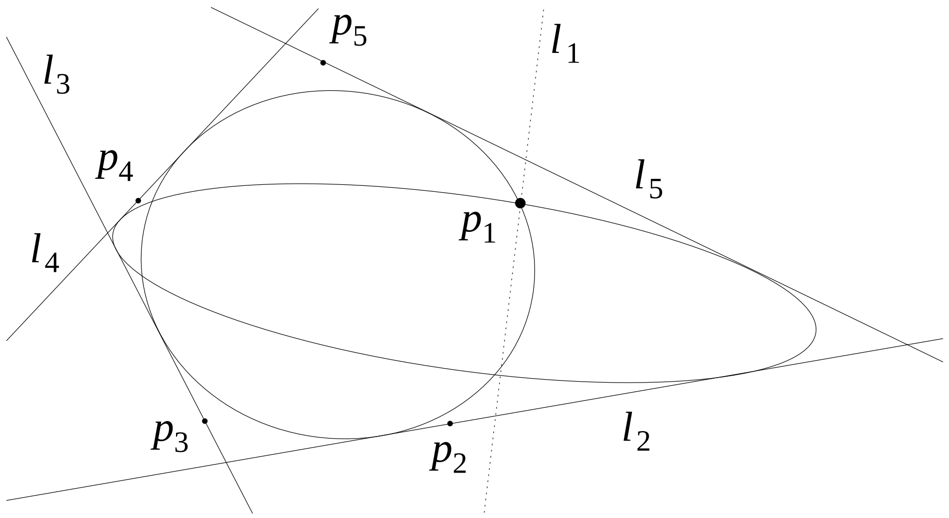
Frank Sottile

Figure 1 in §4 is based upon the configuration of points and lines of Figure 10 of "The number of conics tangent to five given conics : the real case"[1]. In §4, we claimed that the configuration was 'maximal': For any choice of j of the points $p_1, \ldots, p_5$, there will be $2^{\min\{j,5-j\}}$ real conics incident upon the points and tangent to the $5-j$ lines which do not contain the points. In this supplementary note, we consider the 8 different (up to rotational symmetry) possible choices of j points. For each, we illustrate the (real) conics which are incident upon the points and also tangent to the lines.

First, since the configuration is centrally symmetric, there is a circle passing through all five points (dotted curve) and a circle tangent to all five lines (solid curve).

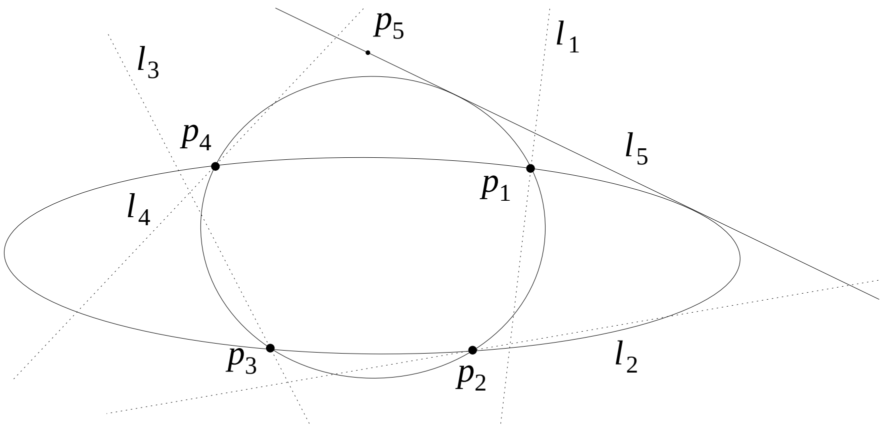


There is one way to choose a single point and there are two conics incident upon a single point and tangent to four lines:

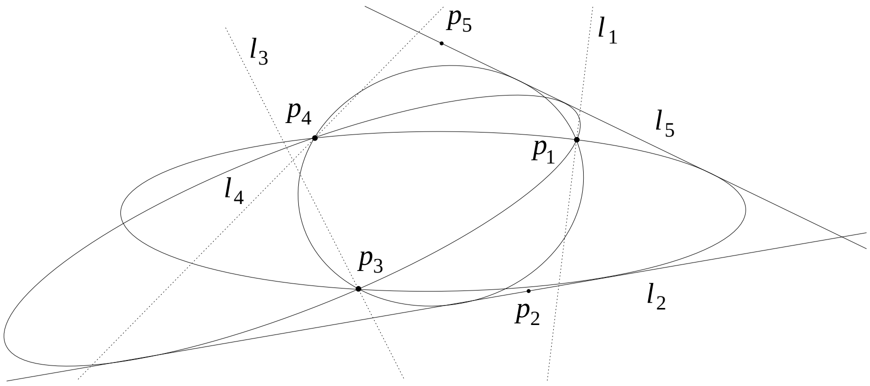


[1]Manuscript by F. Ronga, A. Tognoli, and Th. Vust, available at http://www.unige.ch/math/, 1995.

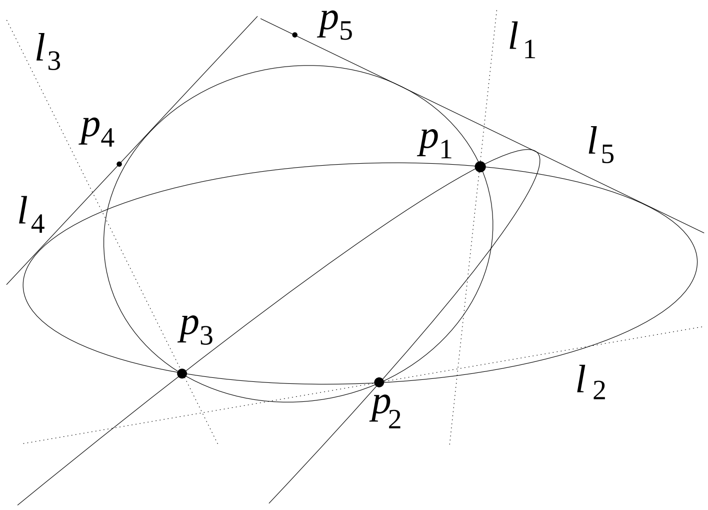
Similarly, there is one way to choose four points:



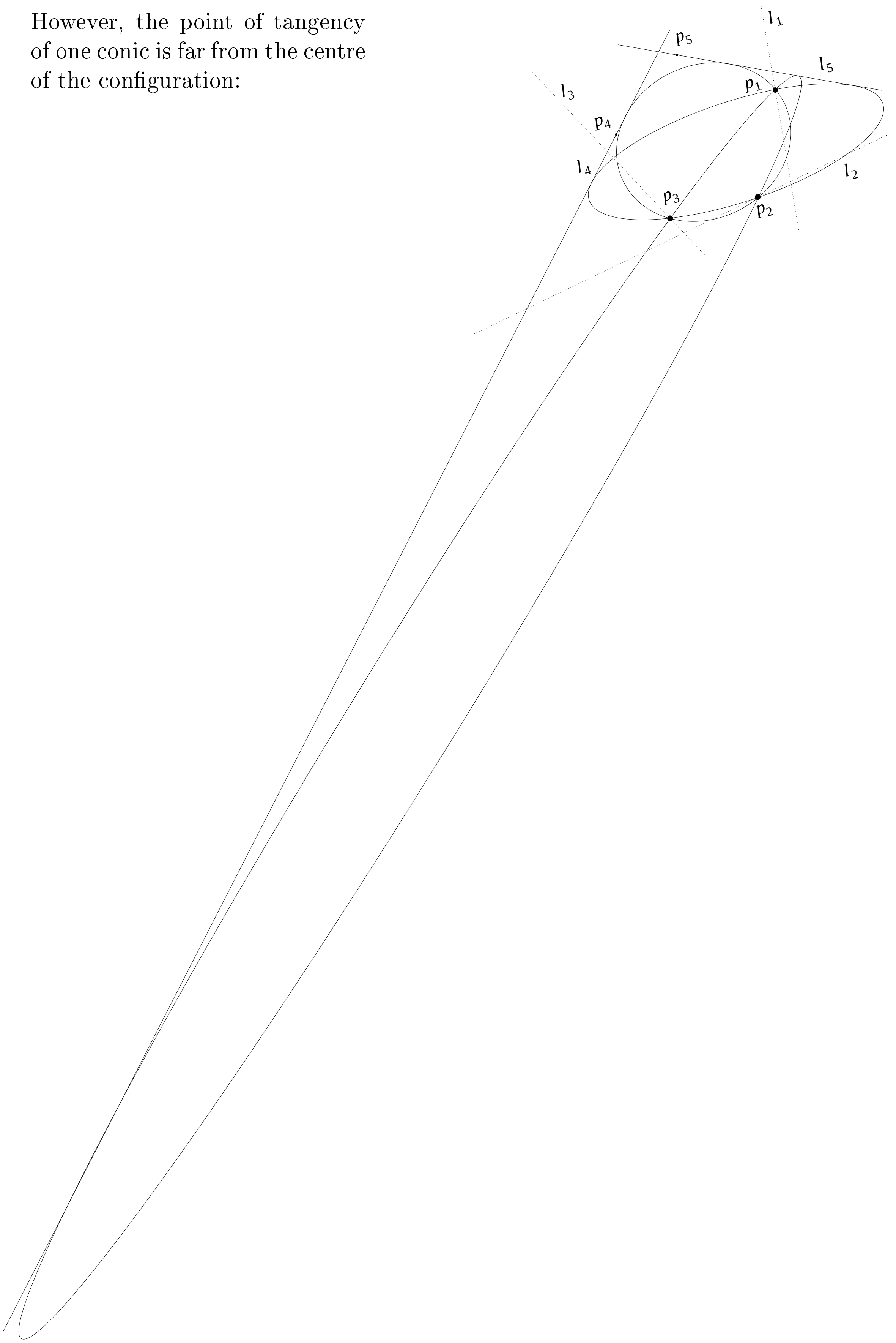
There are two different ways to choose three points. With each way, we can easily draw three of the four conics. Note that this implies that the fourth (not illustrated) is real: The cycle consisting of all four conics is a real zero-cycle and real zero-cycles have an even number of complex points.



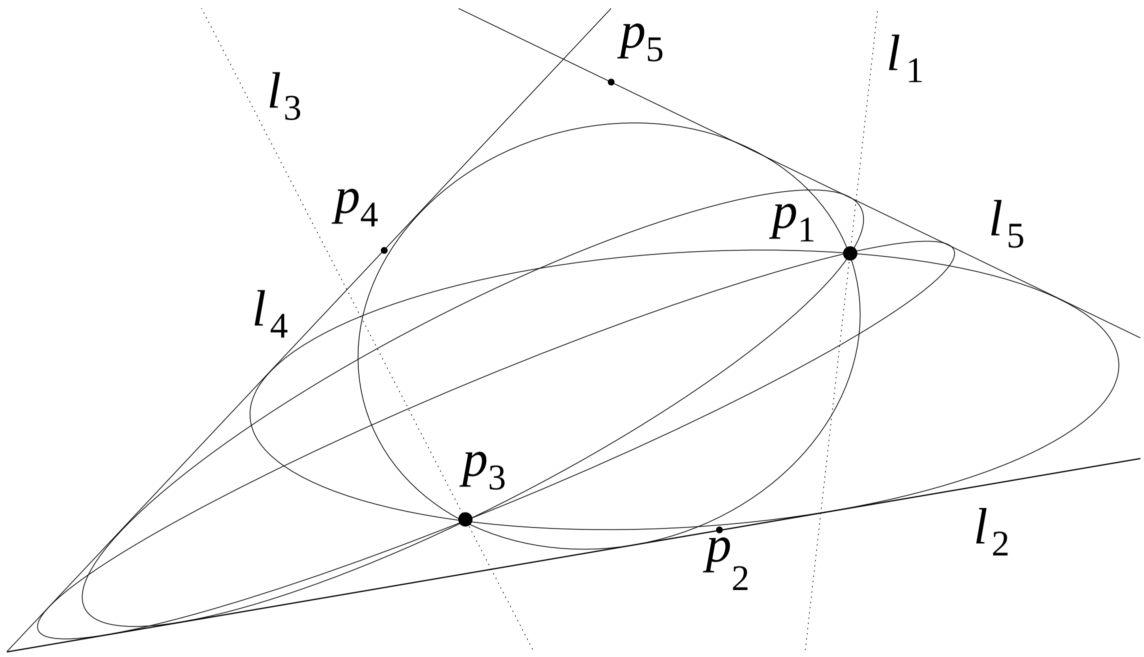
For the other, we may similarly draw three of the four conics.



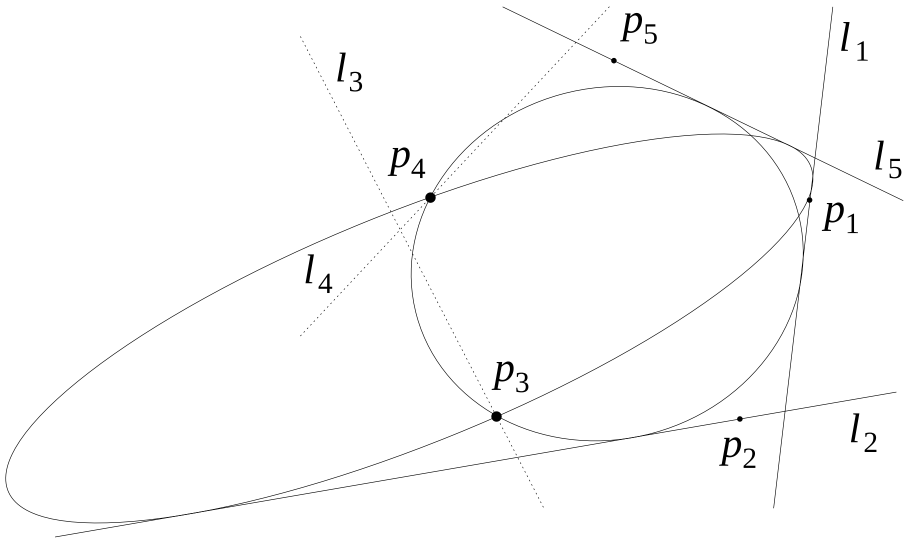
However, the point of tangency of one conic is far from the centre of the configuration:



There are two ways to choose two points. For one, we are able to draw all four conics. (This is Figure 1 of "Enumerative Geometry for Real Varieties").



For the final configuration, we are presently unable to draw even three of the four conics. However, the arguments of Ronga, et. al. (in §3 of their manuscript) show that there are four real conics passing through the two points p_3 and p_4 and tangent to the three lines l_1, l_2, and l_5.



We remark in closing that these pictures were drawn by trial an error using the programming language encapsulated post script. It should be possible to use some mathematical software to solve the equations and fill in the gaps in these pictures.
— Frank Sottile