

Proposal for IMA Participating Institutions Graduate Student Summer Program

Title: Applicable Algebraic Geometry

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LOCATION AND DATES:

We propose to hold an IMA PI Graduate Student Summer Program on the campus of Texas A&M University in College Station, Texas, running from Monday, July 23 through Friday, August 10 in 2007. Travel days are July 22 and August 11. These dates are to accommodate both the end of the 2006-2007 IMA thematic year on Applications of Algebraic Geometry and the academic schedule at the Technische Universität Berlin.

BRIEF PROGRAM DESCRIPTION:

Recent years have seen applications of many ideas and techniques from algebraic geometry to problems in applied mathematics and engineering. Part of this is a recognition of essential algebraic structures in applied problems and part is a need in applications for exact/certifiable results. It is also due in no small measure to modern, simplified presentations of algebraic geometry, interest in particular examples, and the growing use of computers in algebraic geometry. The 2006-2007 IMA Thematic Year "Applications of Algebraic Geometry" will showcase these trends and lead to further, deeper applications of algebraic geometry. The purpose of this IMA PI Summer Graduate Program will be to prepare the ground for the future by introducing graduate students from IMA participating institutions to some of these exciting developments and new perspectives.

Holding this Graduate Student Program in the summer after the IMA thematic year will ensure that its curriculum incorporates the most current and interesting trends in the applications of algebraic geometry. Hosting it in the Department of Mathematics at Texas A&M will allow this Graduate Student Summer Program to take advantage of the many A&M faculty who have interests in these applications of algebraic geometry and who have expertise in organizing conferences and instructional workshops.

The program will be structured around a course in applicable algebraic geometry, treating foundational material as well as current applications. The foundations will include Gröbner bases, toric varieties, and real algebraic geometry, while the applications will be drawn

from optimization, non-linear computational geometry, algebraic statistics, and mathematical biology. We will emphasize computational issues by including computer tutorials and laboratories on relevant software. We will also have guest lectures explaining current research topics. This multi-tiered menu, ranging from introductory material through current research, will ensure that every student gains something from their experience.

OUTLINE OF PROGRAM:

The core of this summer Graduate Program will be two series of lectures. One by Sottile and Theobald will consist of 25 lectures, provide foundational background, and also cover current applications in optimization and in non-linear computational geometry. Serkan Hoşten and Seth Sullivant will give a joint series of 6-10 lectures on algebraic statistics and applications to biology. Both lecture series will include regular assigned problems and a joint daily discussion section, possibly run by postdoc mentors. These mentors will include Texas A&M Postdocs Luis Garcia (who works in algebraic statistics, mathematical biology, and geometric modeling) and Chris Hillar (an NSF postdoc working on real solutions to polynomial equations). We may also invite some IMA postdocs to participate in the program.

Many applications of algebraic geometry are facilitated by computer experimentation, calculation, and user-friendly software. Because of this, some of the problems from the course will include computer work, in levels varying from the simple computation of examples, to full-blown computer laboratory projects. To familiarize the students with this aspect of the course, we will hold regular tutorials on installing and using relevant mathematical software, (such as MAPLE, SINGULAR, MACAULAY 2, SOSTOOLS, and GLOPTYPOL).

Since our goal is applications, we will have a guest lecture series, in which we invite 4-8 experts to each give a 2-part lecture on some application of algebraic geometry in their work. While some of the guest lecturers will be professors at Texas A&M, we may also invite outside experts. Currently, we have asked Texas A&M Professors Schenck (splines or coding theory), Rojas (complexity of real computation), Matusevich (algebraic geometry and special functions), and Keyser (Geometric Modeling). We expect to invite others drawn from among the visitors to the IMA thematic year.

DETAILS OF COURSE:

The lecture series of Sottile and Theobald will consist of five parts, each taking roughly 5 lectures. The first three parts are foundational, the last two are advanced topics.

- I. Quick introduction to basic concepts from algebraic geometry that will be used throughout the course. Projective and affine varieties, ideals, Gröbner bases, and standard examples.
- II. Basic ideas and algorithms from real algebraic geometry. Sturm sequences and Positivstellensatz, real solutions to polynomial equations, and enumerative real algebraic geometry. This prominence of real algebraic geometry is because in applications, real solutions are often much more important than complex ones.
- III. Deformation and numerical techniques in algebraic geometry. Introduction to toric varieties, Kushnirenko's and Bernstein's theorems, deformation techniques for polynomial equations, and polyhedral homotopy method.
- IV. Connections between real algebraic geometry (positivity of polynomials) and semi-definite programming in optimization. Schmüdgen's and Putinar's Theorems, sum-of-squares relaxations of problems in class NP, and convergence to optimal solutions.

- V. Algebraic geometry in nonlinear computational geometry and geometric modeling. Stewart platform and other kinematic manipulators, line problems, and the representation and implicitization of Bézier curves and surfaces.

Sottile and Theobald are currently developing a graduate textbook on Applicable Algebraic Geometry. The lectures in this series would be based upon some parts of that book, and participants will be provided with copies of the text.

The lecture series of Hoşten and Sullivant will consist of 6-10 lectures, delivered in the last two weeks of the Graduate Program. Part will be on algebraic statistics, which uses algebraic geometry for making statistical inferences, as many statistical models for discrete random variables are classical algebraic varieties. The course will explain this connection and discuss some of the interesting geometry, as well as the statistical consequences of the algebraic analysis, such as in maximum likelihood estimation. The second part of the course will be in computational biology, particularly on the relevance of algebraic statistical models to genome sequence analysis.

A common theme in these applications is the role of symbolic computation and of classical and concrete algebraic varieties, such as toric varieties. Also, although this is not apparent from the course descriptions, geometric combinatorics, particularly polytopes and vector configurations, play a fundamental role. Throughout the lectures of both courses, there will be an emphasis on symbolic computation, classical varieties, and the role of geometric combinatorics in applicable algebraic geometry.

LECTURERS:

Sottile and **Theobald** are two organizers of the IMA workshop on Non-linear Computational Geometry, scheduled for May/June of 2007. Both have prior experience organizing large-scale scientific meetings, and giving short advanced courses. In November 2003, Sottile co-organized a Oberwolfach-Seminar on computational algebraic geometry (a short course for European students), he chaired the organization of the 2004 MSRI semester on Topological Aspects of Real Algebraic Geometry, and he gave a short course on “Real Solutions to Equations from Geometry” in November 2005 at the Institut Henri Poincaré. (Notes are available at www.math.tamu.edu/~sottile/pdf/IHP.pdf.) Theobald is giving a 3-week graduate course on semidefinite programming methods in Berlin this May as part of a graduate school on Optimization Methods in Discrete Geometry which he is helping to organize.

Hoşten and **Sullivant** are two organizers of the IMA workshop on Applications in Biology, Dynamics, and Statistics, scheduled for March 2007. Hoşten is an associate professor at San Francisco State University and an experienced organizer. Sullivant is giving a short course on algebraic statistics in June of 2006 at the Sophus Lie Centre in Norway. He is currently a Junior Fellow at Harvard, and already a leader in algebraic statistics.

PROGRAM:

The two courses will have a total of 35 one-hour lectures, and there will be 6-8 guest lectures of 2 hours each. This will result in four hours each day of classroom instruction and lectures. We expect also to schedule one hour each day for a discussion session and one hour each day for a computer lab. This program will be organized around coffee breaks, and the lecturers will be available outside of lectures.

FACILITIES:

The lecture rooms, a common room, and the mathematics department computer labs are all located in Blocker Hall, a modern building which also houses the mathematics department. It is very close to the conference housing and to North Gate, the commercial district adjacent to the University.

Blocker Hall has a variety of computer laboratories, including five CalcLab rooms with varying capacities from 24 to 56 student computer workstations and two University open access computer classrooms, with 56 student computer workstations each. Since these computer labs will be used during the summer for regularly scheduled classes, we will book the lab (or labs) whose capacity is closest to the number of participants. Students will have computer accounts, and we will make provisions for students who bring their own laptops. In fact, we shall encourage students to bring their own computers. Our experience at previous short courses, particularly the Oberwolfach Seminar which Sottile helped to organize and Theobald attended, demonstrated that having participants bring their own computers increases the level and intensity of the activities.

EXPERIENCE IN RELATED PROGRAMS:

We plan to take full advantage of the extensive experience that the staff and faculty in the Department of Mathematics at Texas A&M possess at running summer programs for undergraduate students. For instance, every summer for the past six years the Mathematics Department has run a very successful REU program with 20 students divided into two or three groups. Indeed, the REU grant at Texas A&M has just been renewed. Moreover, the Department is also very experienced at running regular conferences, including a recent IMA-funded conference on Geometric Group Theory and a yearly summer conference in Linear Analysis.

In addition to the staff and faculty expertise this indicates, we would like to benefit from the REU program itself, by holding some joint scientific activities between the REU programs and the Graduate Summer school. This would be to the advantage of the participants in both programs.

In May of 2002, our department hosted a CBMS conference featuring Bernd Sturmfels who lectured on “Solving Polynomial Equations” (now a book). It was preceded by a short course for graduate students to bring them up to the level of the main lectures, where the lectures were given by members of our faculty. At that meeting, there was an extremely successful computer lab involving installing and running some mathematical software, in which both Sottile and Theobald participated. This was the genesis of our plan to have significant, hands-on-computer work as part of our Summer Graduate Program.

This May 2006, our department will organize another school for graduate students and recent Ph.D.s’. Professor John Little of Holy Cross College will give a 3-day short course on algebraic geometry codes, as part of the Texas Algebraic Geometry Seminar, a yearly weekend meeting of algebraic geometry faculty, postdocs, and students from Texas.

Sottile will spend \$5,000 from his NSF CAREER grant on the proposed Summer Graduate School. This will be used where it is most needed, possibly on additional (non Participating Institution) students, post-doc mentors, or senior visitors for the guest lectures.

The Department of Mathematics will contribute \$2000 towards defraying the cost of refreshments and other logistical expenses, and is requesting a matching contribution of another

\$2000 from the College of Science with this same goal. In addition, staff members will assist with logistics and refreshments.

The algebraic geometry group at Texas A&M has a pending NSF RTG grant with a focus on applications of algebraic geometry. It is currently in the second round of review. Whether or not it is funded, we have a deep interest in training activities such as this Summer School. This group includes Sottile and Matusевич, as well as H. Schenck, P. Stiller, J.M. Landsberg, J.M. Rojas, P. Lima-Filho, postdocs L. Garcia and C. Hillar, and Computer Science faculty member John Keyser.

SOCIAL PROGRAM:

We will host a welcome reception and barbecue on the first Monday, July 23rd, either at the home of a faculty member or at a park. (This is traditional for our summer REU's.) Similar activities will be planned for subsequent weeks, and lecturers and post-doc mentors will also attend the social activities. A number of Texas A&M graduate students and some of our postdocs will also take part in and assist with this IMA PI Summer Graduate Program. Their presence will help to enliven the program.

This Summer Graduate Program will coincide with most on-campus REUs, and the Texas A&M Graduate Studies office organizes many activities for the REU students across the campus. These include tours, talks, and some social events. The students in this summer Graduate program would be invited to participate in many of these events.

HOUSING:

The participants will stay in on-campus conference housing which is also used for summer REU programs. Among the amenities offered are community kitchens, pool and fitness center, aerobics room, multimedia room, big-screen TV lounges and surround sound movie theater. There are also individual and group study rooms with ethernet, and ten computer study rooms. More information is available at <http://www.livethetradition.com/>.