Tensors: Discriminants and Hyperdeterminants
Second Session with Bernd Sturmfels at Ibadan

The *hyperdeterminant* of a tensor is a natural generalization of the determinant of a square matrix. How is it defined? We shall explore this concept and discuss what it means for symmetric tensors. A tensor has *rank 2* if it is the sum of two tensors of rank 1, but it does not have rank 1. A tensor has *rank 3* if it is the sum of three tensors of rank 1, but it is not the sum of two tensors of rank 1, etc... This notion depends on the field: the rank over $\mathbb{C}$ can be smaller than the rank over $\mathbb{R}$.

Question 1: What is the complex rank of the Ibadan tensor u?

Question 2: What is the real rank of the Ibadan tensor u?

Question 3: How can we distinguish $2{\times}2{\times}2$ tensors of real rank 2 from those of real rank 3?

Question 4: Let A and B be 3×3-matrices of variables, and consider the univariate polynomial $f(t) = \det(A+tB)$. The discriminant of $f(t)$ is a polynomial in 18 variables. Compute it explicitly.

Question 5: Determine the hyperdeterminant of format $2 \times 3 \times 3$.

Question 6: How many distinct entries does a symmetric tensor of format $n \times n \times n$ have?

Question 7: Determine the restriction of the $2 \times 2 \times 2$ hyperdeterminant to symmetric tensors.

Question 8: Determine the restriction of the $3 \times 3 \times 3$ hyperdeterminant to symmetric tensors.

Question 9: The discriminant of a plane cubic is a polynomial in 10 unknowns. What is its degree? Compute this discriminant explicitly. How many monomials does its expansion have?

Question 10: Determine the restriction of the $2 \times 2 \times 2 \times 2$ hyperdeterminant to symmetric tensors.