Mangi Problems

Let $I \subset R[x_1, .., x_n] =: R$, if $f \in R$, is $f \in I$? Is $V(I) \subset V(f)$?

1. Determine if $f \in I$ given
$$(i)\ f = x^3 - 1$$
$$I =< x^6 - 1, x^5 + x^3 - x^2 + 1 >$$
$$(ii)\ f = x^5 - 4x + 1$$
$$I =< x^3 - x^2 + x >$$
Let us do it $a)$ by hand $b)$ by Macaulay2.

2. Order the following using LEX, GRLEX, GREVLEX, $f \in R[x, y, z]$
$$(i)\ 9x^3y - \frac{1}{7}xy^2z + xyz^3 - xy^4$$
$$(ii)\ xyz^4 - 5yz^5 + x^3y^3 + x^2y^4$$
$$(iii)\ 2x^3y^5z^2 - 3x^4yz^5 + xyz^3 - xy^4$$
Give $LT(f) = in(f)$, $LM(f)$, $multideg(f)$

3. Determine the monomial order used for each of the following
$$(i)\ 7x^2y^4z - 2xy^6 + x^2y^2$$
$$(ii)\ xy^3z + xy^2z^2 + x^2z^3$$
$$(iii)\ x^4y^5z + 2x^3y^2z - 4xy^2z^4$$

4. Determine a remainder on division of the polynomial f by order
$$(i)\ f = x^7y^2 + x^3y^2 - y + 1$$
$$F = \{xy^2 - x, x - y^2\}$$
$$(ii)\ xy^2z^2 + xy - yz$$
$$F = \{x - y^2, y - z^3, z^2 - 1\}$$